A New Application of the Fixed-Points Theory for the Control of Kinetic Energy of a Continuous Structure

Jedol Dayou[†]

Vibration and Sound Research Group (VIBS), School of Science and Technology, University of Malaysia Sabah, Locked Bag 2073, 88999 Kota Kinabalu, Sabah, Malaysia

(Received 30 August 2004; accepted 11 November 2005)

The fixed-points theory was originally used to determine the optimum tuning and damping ratios of a vibration neutraliser that may flatten the frequency response function (FRF) of a simple structure. Although the theory has also been used for the control of vibration of a continuous structure, applications have been limited only to point control. In this paper, a new application of the theory is discussed, which is for the global control of vibration of a continuous structure. The theory is used to determine the optimum tuning and damping ratios of the vibration neutraliser that flatten the global response of the structure. Kinetic energy is used as a measure of the structure's global response, and the application is demonstrated on undamped and damped simply supported beams. It is found that by using these optimum values, it is possible to remove the effects of the dominant mode leaving only the effect of the residual modes in the global behaviour. This shows that the theory can also be used to reduce the vibration of a continuous structure at all points instead of at a particular point only, such as in the conventional application.

† Member of the International Institute of Acoustics and Vibration (IIAV)

1. INTRODUCTION

A new application of the fixed-points theory is discussed. The theory is used to determine the optimum tuning and damping ratios of the vibration neutraliser that can be used to flatten the global behaviour of a continuous structure for both undamped and damped cases.

The vibration neutraliser has been used in many applications since it was invented in 1909.[1,2] Applications range from small handheld tools, such as electrical clippers,[3] to giant structures such as high-rise buildings[4,5] and bridges,[6] to large moving structures, such as aircraft.[7,8,9] The vibration neutraliser was designed in such a way that its natural frequency coincides with either 1) the problematic resonance frequency of the host structure or 2) the frequency of disturbance away from the resonance region.[10] In either case, the vibration neutraliser becomes less effective and may even increase the vibration amplitude of the host structure when there is a change in the frequency of the excitation force.

Previously, the fixed-points theory was used to determine the optimum tuning and damping ratios of a vibration neutraliser that flatten the frequency response function (FRF) of a simple structure. The theory has its roots dating back to 1932 when Hahnkamm[11,2] suggested that the desired natural frequency of the vibration neutraliser should be given by

$$f_{n_{opt}} = \frac{f_h}{1+\mu}, \tag{1}$$

where $f_{n_{opt}}$ is the neutraliser's optimal natural frequency, f_h is the host's resonance frequency, and μ is the ratio between the neutraliser's and the host's mass. Later, Brock[12] derived the expression for the neutraliser's optimal damping ratio, which can be written as

$$\zeta_{n_{opt}} = \sqrt{\frac{3\mu}{8(1+\mu)}}, \tag{2}$$

where ζ_n is the neutraliser's damping ratio defined as $\zeta_n = c_n/(2M_n f_n)$; c_n, M_n, and f_n are the neutraliser's damping, mass, and natural frequency, respectively. Using these optimum values, the FRF curve of the composite system can be made relatively flat with no new resonance occurring.

The fixed-points theory was found to be very useful and has been used as one of the design laws when fabricating a vibration neutraliser.[13] However, the applications of the theory have been limited only to relatively simple structures. The theory has also been used for continuous structures with well-separated natural frequencies, such as beams, but only in regards to point control problems.[14] These are the conventional applications of the theory.

In this paper, the application of the theory is extended to a new area, which is for the control of global responses of a continuous structure. Kinetic energy is used as a measure of the global responses, and the theory is applied to determine the optimum tuning and damping ratios of the control device. It is found that the theory can be used to derive the optimum value of these parameters that flatten the FRF of the structure in terms of its kinetic energy. Therefore, it is shown that the theory can also be used to reduce the vibration of a continuous structure at all points instead of only at a particular point as in conventional applications. The new application is demonstrated on a simply supported beam as a host structure, for both undamped and damped cases, through some computer simulations.

2. PROBLEM FORMULATIONS

It is well known that for the case of continuous structures, reducing the vibration amplitude at one point may increase the amplitude at some other points.[15] This is true unless a global measure is taken where the aim is to reduce the overall vibration at all points. In this paper, the kinetic energy of the structure is used to measure the structure's global response.

The time-averaged kinetic energy of a relatively simple structure, such as a beam, can be written as[15,16]

$$KE = \frac{M_s\omega^2}{4}\mathbf{q}^H\mathbf{q}, \tag{3}$$

where KE is the kinetic energy, M_s is the modal mass of the structure, and ω is the circular frequency of the primary force; whereas $\mathbf{q}$ is the M-length vector of the modal displacement amplitude of the structure. The vector of the modal displacement amplitude is given by $\mathbf{q} = \mathbf{A}\mathbf{g}$, where $\mathbf{A}$ is the $M \times M$ diagonal matrix of the complex modal displacement amplitude whose elements are given by

$$A_m = \frac{1}{M_s(\omega_m^2 - \omega^2 + i2\zeta_m\omega_m\omega)}, \tag{4}$$

and $\mathbf{g}$ is the M-length vector of the generalised force acting on the structure. In Eq. (4), ζ_m is the modal damping ratio, and ω_m is the m-th resonance frequency of the structure, and i is the imaginary number given by $\sqrt{-1}$.

If a vibration neutraliser is fitted as a control device, then there are two contributing forces that make the generalised force, g: 1) a primary uncontrolled force and 2) the feedback force generated by the neutraliser. Therefore, the modal displacement amplitude is modified to

$$\mathbf{q} = \mathbf{A}(\mathbf{g}_p + \Phi_k \mathrm{f}_k), \tag{5}$$

where $\mathbf{g}_p$, Φ_k, and f_k are the vectors of the generalised primary force, the normalised mode shape of the structure evaluated at the neutraliser's location, and the amplitude of the feedback force from the neutraliser, respectively.

The feedback force from the neutraliser can be written as[17]

$$\mathrm{f}_k = -K_k w(x_k), \tag{6}$$

where $w(x_k)$ is the displacement of the structure at location x_k, which is given by

$$w(x_k) = \Phi^T\mathbf{q} = \sum_{m=1}^{M}\phi_m(x_k)q_m, \tag{7}$$

and K_k is the dynamic stiffness of the neutraliser given by

$$K_k = -\omega^2 M_k\left[\frac{1 + i2\zeta_k(\omega/\omega_k)}{1 - (\omega/\omega_k)^2 + i2\zeta_k(\omega/\omega_k)}\right], \tag{8}$$

where M_k, ζ_k, and ω_k are the mass, damping ratio, and resonance frequency of the neutraliser, respectively. The damping ratio of the neutraliser is defined as $\zeta_k = C_k/(2M_k\omega_k)$, where C_k is the neutraliser's critical damping, whereas, the resonance frequency is $\omega_k = \sqrt{k_k/M_k}$ and k_k is the neutraliser's stiffness constant. Combining Eqs. (6) and (7) gives the neutraliser's feedback force in terms of the modal amplitudes of the structure:

$$\mathrm{f}_k = -K_k\Phi^T\mathbf{q}. \tag{9}$$

Combining Eqs. (5) and (9) gives the modal displacement amplitudes, which can be written either as:

$$\mathbf{q} = \mathbf{A}(\mathbf{g}_p - K_k\Phi_k^T\Phi_k\mathbf{q}) \tag{10}$$

or

$$\mathbf{q} = [\mathbf{I} + K_k\mathbf{A}\Phi_k^T\Phi_k]^{-1}\mathbf{A}\,\mathbf{g}_p. \tag{11}$$

The modal displacement amplitudes of the structure in the vicinity of the m-th natural frequency can be approximated well by the structure's m-th mode only. In this situation, Eq. (11) can be written as[18]

$$q_m = \frac{A_m g_{pm}}{1 + K_k A_m \phi_m^2(x_k)}. \tag{12}$$

This is true when the vibration modes are well separated. The dynamic stiffness of the neutraliser in Eq. (8) can also be rewritten as

$$K_k = -M_k\omega^2\left[\frac{k_k + i2\zeta_k\sqrt{M_k k_k}\,\omega}{k_k - M_k\omega^2 + i2\zeta_k\sqrt{M_k k_k}\,\omega}\right]. \tag{13}$$

To reduce the mathematical burden, let us also assume that there is no damping in the structure. Therefore, the complex modal amplitude of the structure can be rearranged as

$$A_m = \frac{1}{(k_m - M_s\omega^2)}, \tag{14}$$

where k_m is the effective bending stiffness of the structure. When combining Eqs. (12)-(14), one obtains

$$q_m = \frac{g_{pm}\left[\left(i2\zeta_k\sqrt{M_k k_k}\,\omega\right) + \{k_k - M_k\omega^2\}\right]}{\left[\begin{array}{l}\left(i2\zeta_k\sqrt{M_k k_k}\,\omega\right)\{k_m - M_s\omega^2 - M_k\omega^2\phi_m^2(x_k)\}\\ +\{(k_m - M_s\omega^2)(k_k - M_k\omega^2) - M_k k_k\omega^2\phi_m^2(x_k)\}\end{array}\right]}. \tag{15}$$

This leads to the following equation for the kinetic energy of the structure in the vicinity of its m-th natural frequency:

$$KE_m = \frac{M_s\omega^2 g_{pm}^2}{4} \times \frac{(2\zeta_k\omega_k\omega)^2 + \{\omega_k^2 - \omega^2\}^2}{\left[\begin{array}{l}(2\zeta_k\omega_k\omega)^2\{k_m - M_s\omega^2 - M_k\omega^2\phi_m^2(x_k)\}^2\\ +\{(k_m - M_s\omega^2)(\omega_k^2 - \omega^2) - M_k\omega_k^2\omega^2\phi_m^2(x_k)\}^2\end{array}\right]}. \tag{16}$$

Rearranging the equation gives

$$KE_m = \frac{\omega^2 g_{pm}^2}{4M_s} \times \frac{(2\zeta_k\omega_k\omega)^2 + \{\omega_k^2 - \omega^2\}^2}{\left[\begin{array}{l}(2\zeta_k\omega_k\omega)^2\{\omega_m^2 - \omega^2 - \mu\omega^2\phi_m^2(x_k)\}^2\\ +\{(\omega_m^2 - \omega^2)(\omega_k^2 - \omega^2) - \mu\omega_k^2\omega^2\phi_m^2(x_k)\}^2\end{array}\right]}, \tag{17}$$

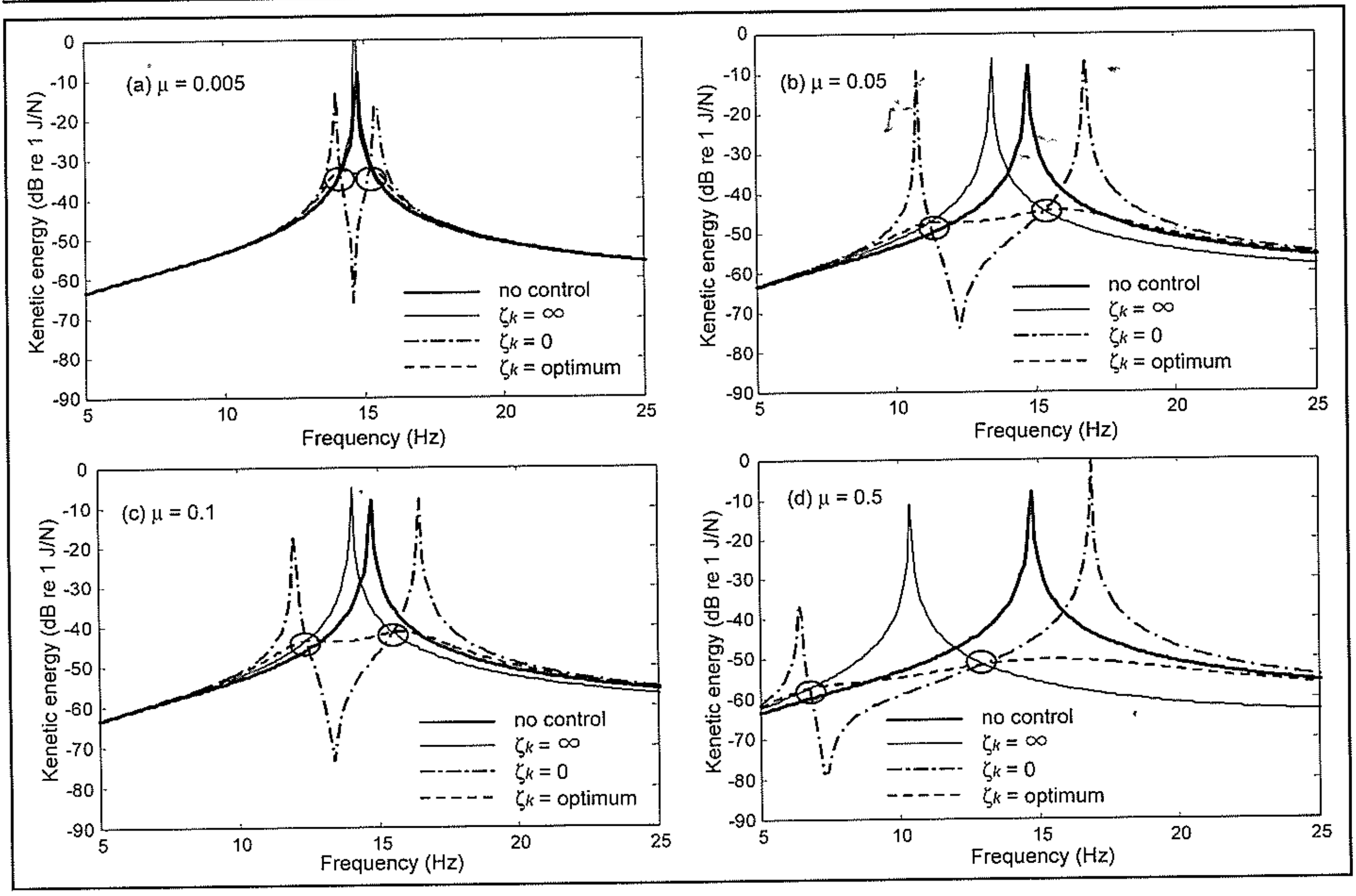

Figure 1. Application of the fixed-points theory in the global vibration control of the undamped simply supported beam. $x_k = 0.5L$ for all graphs, and the control target is the first natural frequency.

where μ is the ratio of the neutraliser's mass to the host structure's mass. Taking

$$\mu = \frac{M_k}{M_s}; \quad f_{mn} = \frac{\omega_k}{\omega_{mn}}; \quad \text{and} \quad g_{mn} = \frac{\omega}{\omega_{mn}}, \tag{18}$$

Eq. (17) can be rearranged as

$$KE_m = \frac{M_s \omega^2 g_{pm}^2}{4k_m^2} \times \frac{(2\zeta_k f_m g_m)^2 + \{f_m^2 - g_m^2\}^2}{\left[\begin{array}{l}(2\zeta_k f_m g_m)^2\{1 - g_m^2 - \mu g_m^2 \phi_m^2(x_k)\}^2 \\ +\{(1-g_m^2)(f_m^2 - g_m^2) - \mu f_m^2 g_m^2 \phi_m^2(x_k)\}^2\end{array}\right]}. \tag{19}$$

The kinetic energy in Eq. (19) now has a similar form to the displacement amplitudes of a simple primary structure in the conventional theory and can be rewritten as

$$KE_m = \frac{M_s \omega^2 g_{pm}^2}{4k_m^2}\left(\frac{A^2\zeta_k^2 + B^2}{C^2\zeta_k^2 + D^2}\right), \tag{20}$$

where

$$A = 2f_m g_m;\ B = g_m^2 - f_m^2;\ C = 2f_m g_m\{g_m^2 - 1 + \mu g_m^2 \phi_m^2(x_k)\};$$

and

$$D = \mu f_m^2 g_m^2 \phi_m^2(x_k) - (g_m^2 - 1)(g_m^2 - f_m^2). \tag{21}$$

Therefore, the procedures in the conventional fixed-points theory can be followed to obtain the optimum tuning and damping ratios of the vibration neutraliser which are given, respectively, as[19,20]

$$f_{m_{opt}} = \frac{1}{1 + \mu\phi_m^2(x_k)} \tag{22}$$

and

$$\zeta_{k_{opt}} = \sqrt{\frac{3\mu\phi_m^2(x_k)}{8(1 + \mu\phi_m^2(x_k))}}. \tag{23}$$

It can be seen that Eqs. (22) and (23) have a similar form to the neutraliser's optimum tuning and damping ratios in the conventional fixed-points theory. Some of the characteristics regarding the optimum tuning and damping ratios in Eqs. (22) and (23) were discussed by the author elsewhere.[20]

3. COMPUTER SIMULATION AND DISCUSSION

Equations (22) and (23) are the optimal or the desired tuning and damping ratios of the vibration neutraliser, respectively, when the device is used for the control of the kinetic energy of a continuous structure. The equations were derived based on the procedure developed in the conventional fixed-points theory. In this section, the effects of the application of the optimum vibration neutraliser are presented. For this purpose, an undamped simply supported beam was selected as a host structure with the following properties: physical dimensions $= 1 \times 0.0381 \times 0.00635$ m; material density $= 7870$ kg/m^3; Young's modulus $= 2.07 \times 10^{11}$ N/m^3; and a unity amplitude of a primary harmonic force applied at $0.1L$ ($x_f = 0.1L$). The simply supported beam used in this example is a beam that has no displacement at its supports but is allowed to have a slope at these supports. Such beams are also known as hinged or pinned beams.

In this study it was found that there are two fixed points in the global response of a continuous structure when a vibration neutraliser is used as a control device.[19] The next step taken was to show that the fixed-points theory can be used to determine the optimum resonance frequency and damping ratio of the neutraliser that may flatten the kinetic energy of the host structure, as in the conventional theory. This is shown in Fig. 1 for different values of the neutraliser's mass ratio with the beam's first natural frequency as a control target. It can be clearly seen that the two fixed points, P and Q, are present. It can also be seen that the kinetic energy curve is relatively smooth when the neutraliser's tuning and damping ratios are optimised. Apart from that, no new resonance occurs on above and below the beam's natural frequency. This is the desirable result when using the fixed-points theory.

As the neutraliser's mass increases in Fig. 1, the reductions in the kinetic energy also increase. At some points, the effect from the dominant mode is almost removed leaving only the effect from the residual modes. This can be seen in Fig. 1(d), in which the neutraliser's mass ratio $\mu = 0.5$, where a reduction as high as 40 dB can be observed at the resonance frequency, producing a nearly flat kinetic energy curve. The same observation can be shown for higher modes. Discussion on some other aspects regarding this theory can be found in reference.[20]

4. APPLICATION OF THE FIXED-POINTS THEORY FOR A DAMPED STRUCTURE

In the previous section, the application of the fixed-points theory was discussed with an undamped simply supported beam as a host structure. However, in reality, there is always damping in the structure. It is, therefore, interesting to investigate if the theory can be used in real applications when damping is present. For this purpose, the same simply supported beam is used as a host structure but with a small damping ratio of 0.005. This is a practical example since damping in structures is normally small.

The application of the theory to the damped simply supported beam is shown in Fig. 2. The target resonance frequencies are the second (Fig. 2(a)) and third (Fig. 2(b)) modes. The figures show that the kinetic energy of the beam is nearly flat with no new resonance occurring in the frequency range of interest. As the mass ratio is increased, the kinetic energy is reduced to a level where the resonance effects from the dominant mode are almost completely removed. This is quite similar to the observation made in the previous section. This shows that the theory can be used not only for the control of undamped continuous structures but also for damped ones.

5. CONCLUSION

In this technical note, a new application of the fixed-points theory is discussed. In the past, the theory was used to determine the optimum tuning and damping ratios of the vibration neutraliser that can be used to flatten the FRF of a simple host structure. The theory has also been used for vibration control of a continuous structure, but only for point control purposes. The new application discussed in this technical note is for the control of the global behaviour of a continuous structure, which is the control of its kinetic energy. A simply supported beam was used as a host structure in the discussion. It was shown that the theory could be used to determine the optimum tuning and damping ratios of the control device that may flatten the kinetic energy curve of the host structure in the resonance region. This means that the theory can also be used to reduce the vibration of continuous structures, such as beams, at all points instead of at just one particular point as in conventional applications. This is true for both undamped and damped structures with small damping coefficients.

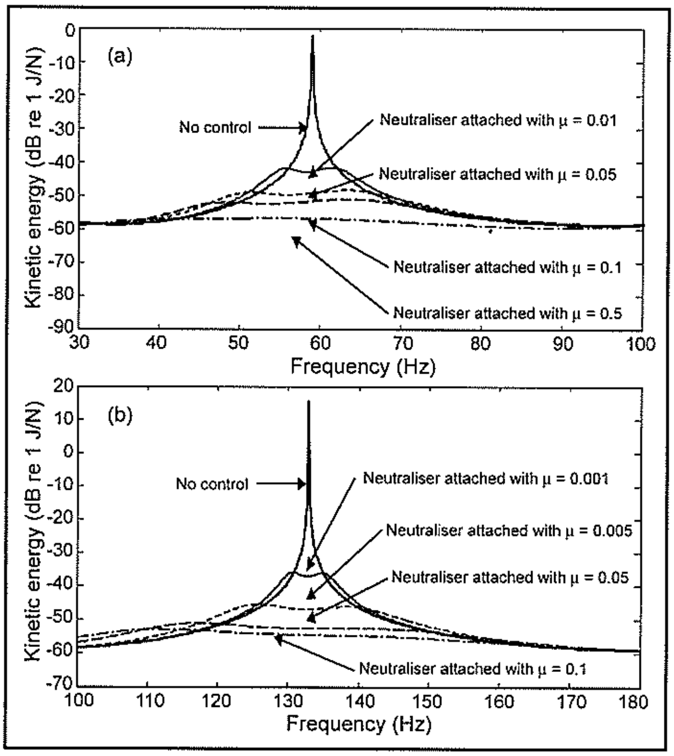

Figure 2. Application of the fixed-points theory on the damped simply supported beam. The modal damping of the beam is $\zeta_m = 0.005$. (a) The control target is the second natural frequency and the vibration neutraliser is applied at $x_k = 0.25L$. (b) The control target is the third natural frequency and the vibration neutraliser is applied at $x_k = 0.5L$.

Acknowledgements

The work presented in this paper has been supported by the Ministry of Science, Technology and Innovation, Malaysia under IRPA research grant No. 09-02-10-0048-EA0046. The author gratefully acknowledges this support.

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