December 2003
Vol. 24 Nos. 3 & 4

PP 2877/11/2004
ISSN 0128-0333

JURNAL FIZIK MALAYSIA

MALAYSIAN INSTITUTE OF PHYSICS

Determination of the optimal tuning ratio of a tunable vibration neutralizer using quadratic minimization technique

Jedol Dayou
School of Science and Technology, University of Malaysia Sabah, Locked Bag 2073, 88999 Kota Kinabalu, Sabah, Malaysia

(Received 24 February 2003)

A technique to determine the optimal tuning ratio of a vibration neutralizer that minimizes the global response of a structure is discussed in this paper. The technique, which is the well-known quadratic minimization, has been previously used for the determination of the optimal amplitudes and phases from secondary forces that minimizes the global response of a structure in active control method. This paper describes how to adopt the technique so that it can be used in optimizing a passive control device such a neutralizer. It is proven that by adopting the quadratic minimization technique, vibration neutralizer can be made as effective as active control device for global control of a structure.

I. INTRODUCTION

Structural vibration is one of the major problems in our modern world especially in the manufacturing industries. This is because of the resulting unpleasant motion and the dynamic stresses, which may lead to fatigue and failure of a structure or machine, and the energy losses and reduction in performance which accompany vibration [1]. In early attempts to deal with the problem, the vibration absorber or vibration neutralizer has been one of the most favorable choices. It has proven to be very effective in reducing tonal vibration of machinery, buildings, bridges and many other mechanical systems with relatively low cost [2]. However, nowadays, there is a need for strong lightweight materials especially in the aircraft industry which unfortunately causes more vibration and noise problems than ever. In this situation, the use of vibration neutralizer is said to be unsuitable, as it would involve a large increase in the weight of the whole structure [3]. Furthermore, in the earlier stages of development, the vibration neutralizer has operated at a fixed frequency, and this condition can enhance rather than attenuate vibration of the host structure if the forcing frequency varies even by a small amount. For this reason, structural vibration engineers turned to active control in order to achieve the goal without the penalty of an increase in mass. By using the quadratic minimization technique, the suitable amount of external energy to be supplied into the problematic structure can be determined so that its vibration is minimized. The technique is now well established and has entered the area of practical application, for example in active control of propeller noise in aircraft using structural actuators [4].

Unfortunately, active control, particularly active structural noise control requires a relatively high degree of external energy to be put into the system, and has the potential for instability [5-6]. Moreover, active control is also expensive and requires a large number of control transducers and sensors [6]. The cost and energy requirement of an active control system prohibits its application to a large class of systems [7]. These penalties have forced researchers to find other control alternatives, and one of the solutions is the tunable vibration neutralizer or TVN [6,7]. However, there are difficulties in implementing the control measure using TVN. The main reason among them is that the mathematical model to determine the appropriate parameters of the TVN is yet to be developed for global control purpose. The work in this paper explores the subject of global vibration control using a tunable vibration neutralizer as an alternative to the widely used active control. It should be noted here that only single frequency excitation is considered in this paper. The reason is because to date, TVN is found to be effective only when dealing with such a case. Never the less, single frequency excitation can be found in many examples such as vibration of the body of a running car, or vibration of the passengers cabin induced by the running aircraft propeller. Therefore, the discussion can be very useful in many real problems.

II. REVIEW ON THE FEED-FORWARD ACTIVE CONTROL METHOD

Before we proceeds on the objective of this paper, it is best if the origin of the theory of the quadratic minimization technique is first presented to gain a better understanding on the subject. Many vibration problems involve a structure being excited by external force. The force can be in a physical form such as excitation from a running engine of a car, or in acoustical form such as sound originated from the engine propeller entering the aircraft passengers cabin.

When excited, arbitrary structure will response by making some displacements from its equilibrium position. If the excitation is harmonic, i.e. repeating every certain period of time, it causes the structure vibrates accordingly. The displacement of the structure at any point can be written as

$$w(x) = \mathbf{\Phi}^T(x)\mathbf{q} \tag{1}$$

$w(x)$ is the displacement at location x, $\mathbf{\Phi}(x)$ is the $M \times 1$ vector of the normalized mode shapes and $\mathbf{q}$ is the $M \times 1$ vector of modal displacement amplitudes of the structure respectively. The superscript T denotes the transpose of the vector.

The modal displacement vector is given by:

$$\mathbf{q} = \mathbf{A}\mathbf{g}_p \tag{2}$$

where $\mathbf{A}$ is the $M \times M$ diagonal matrix of complex modal amplitudes and $\mathbf{g}_p$ is the $M \times 1$ vector of generalized primary forces acting on the structure. The m-th component of the complex modal amplitude $\mathbf{A}$ is:

$$A_m = \frac{1}{M_m(\omega_m^2 - \omega^2 + i2\zeta_m\omega\omega_m)} \tag{3}$$

where M_m, ω_m and ζ_m are the modal mass, the m-th circular natural frequency, and the m-th modal damping ratio of the host structure, and ω is the circular frequency of the excitation force whereas i is imaginary number, $\sqrt{-1}$

The above equations shows the derivation of the frequency responses of an arbitrary structure when an external energy is supplied into it by excitation. As has been discussed before, the response of the structure to the excitation is something that causes the problems, and therefore is preferably to be minimized. In order to achieve this goal, additional physical forces, which are better known as secondary forces, are normally used to combat it. This is made possible by using physical actuator derived by electrical power as in active control method.

With the secondary forces in place, the modal displacement vector in Eq. (2) is written as

$$\mathbf{q} = \mathbf{A}(\mathbf{g}_p + \mathbf{g}_s) \tag{4}$$

where $\mathbf{g}_s$ denotes the amplitude of the secondary forces in generalized form which is expressed as

$$\mathbf{g}_s = \mathbf{\Psi}\mathbf{f}_s \tag{5}$$

$\mathbf{\Psi}$ and $\mathbf{f}_s$ are the $M \times J$ matrix of the structural normalized mode shape evaluated at the location of each of the secondary force and the $J \times 1$ vector of the amplitude of the secondary forces respectively.

The dynamic response of the structure in Eq. (1) is evaluated at each point on the structure. Therefore, it does not represent the general behavior of the structure as a whole. Consequently, the dynamic response is usually written in terms of its kinetic energy that describes the global response of the structure under excitation and is expressed as [8]

$$KE = \frac{M_s\omega^2}{4}\mathbf{q}^H\mathbf{q} \tag{6}$$

M_s is the mass of the structure and the superscript H denotes the Hermitian transpose, which is the transpose of the complex conjugate of matrices or vectors respectively. Eq. (6) describes the kinetic energy as a global response, stored in the whole structure under the influence of the primary forces with secondary control forces in place. In order to minimizes the dynamic response of the structure as a whole, Eq. (6) has to be minimizes. This can be achieved using the so-called quadratic minimization procedures described as follows.

Eq. (6) can be expressed in the standard Hermitian quadratic form as

$$KE = \frac{M_s\omega^2}{4}\{\mathbf{f}_s^H\mathbf{G}^H\mathbf{G}\mathbf{f}_s + \mathbf{f}_s^H\mathbf{G}^H\mathbf{d} + \mathbf{d}^H\mathbf{G}\mathbf{f}_s + \mathbf{d}^H\mathbf{d}\} \tag{7}$$

where

$$\mathbf{d} = \mathbf{A}\mathbf{g}_p;\ \mathbf{G} = \mathbf{A}\mathbf{\Psi}. \tag{8a,b}$$

Eq. (6) is minimized if the vector of the secondary forces is optimum and is given by [8]

$$\mathbf{f}_{so} = -[\mathbf{G}^H\mathbf{G}]^{-1}\mathbf{G}^H\mathbf{d}. \tag{9}$$

The corresponding minimized modal displacement vector is therefore can be written as

$$\mathbf{q}_o = [\mathbf{I} - \mathbf{G}[\mathbf{G}^H\mathbf{G}]^{-1}\mathbf{G}^H]\mathbf{d}. \tag{10}$$

Eq. (9) is the amplitude of the vector of the secondary forces that has to be actively generated to reduce the total kinetic energy stored in the structure. It has both the information about the forces amplitude and also the phase in respect to the excitation from the primary forces. It has to be stressed here that only single frequency excitation is considered and therefore the minimization procedures is valid. This is known as feed-forward active control method and illustrates the performance limits of any active control system which is true for a single frequency excitation as the case in this paper.

III. OPTIMIZATION OF THE TVN USING QUADRATIC MINIMIZATION TECHNIQUE

Historically, vibration neutralizer, consists of a mass-spring-damper system attached to a host structure was also known as ***vibration absorber*** or ***dynamic vibration absorber*** depending to the nature of its application [10-12]. In classic applications, the vibration neutralizer normally has a fixed natural frequency. However, a fairly recent development shows that it is possible to change the natural frequency to a desired value, which is normally performed, by changing its stiffness [2,11,13] and the way of achieving this has been reviewed by von Flotow *et al.* [14] and Brennan [15]. It is also reported that the damping ratio can be altered by active means [12]. Thus, the term ***tuneable vibration neutraliser*** or ***TVN*** refers to a vibration neutralizer with a changeable natural frequency and damping ratio.

If we replace the secondary forces in Section II with TVNs, Eq. (5) can be expressed as

$$\mathbf{g}_t = \boldsymbol{\Psi}\mathbf{f}_t \tag{11}$$

where $\mathbf{g}_t$ and $\mathbf{f}_t$ are the generalized form and amplitude of the feedback forces from the TVNs respectively. In matrix form, the vector of the feedback forces can be written as [16]

$$\mathbf{f}_t = -\mathbf{K}\boldsymbol{\Psi}^T\mathbf{q} \tag{12}$$

$\mathbf{K}$ is the $J \times J$ diagonal matrix of the dynamic stiffness of the neutralizers where the j-th component is given by [17]

$$K_j = -M_j\omega^2\left[\frac{1+i2\zeta_j\alpha_j}{1-\alpha_j^2+i2\zeta_j\alpha_j}\right] \tag{13}$$

M_j, ζ_j and α_j are the neutralizer mass, damping ratio and the tuning ratio respectively. The tuning ratio α_j is given by ω/ω_j, and $\omega_j = \sqrt{M_j/K_j}$ which is the natural frequency of the j-th neutralizer.

If Eq. (11) is substituted into Eq. (6), the kinetic energy with TVNs attached can also be written in the standard Hermitian quadratic form as

$$KE = \frac{M_s\omega^2}{4}\{\mathbf{f}_t^H\mathbf{G}^H\mathbf{G}\mathbf{f}_t + \mathbf{f}_t^H\mathbf{G}^H\mathbf{d} + \mathbf{d}^H\mathbf{G}\mathbf{f}_t + \mathbf{d}^H\mathbf{d}\}. \tag{14}$$

Following the same minimization procedure described by Nelson and Elliott [8] for the feed-forward active control method, the value of the required feedback forces that has to be generated by the TVNs that minimizes the kinetic energy of the host structure will have the same form as in Eq. (9), which is

$$\mathbf{f}_{tr} = -[\mathbf{G}^H\mathbf{G}]^{-1}\mathbf{G}^H\mathbf{f}_{so} \tag{15}$$

$\mathbf{d}$ and $\mathbf{G}$ are given in Eq. (8a,b). It should be mentioned here that the word **required** is used because the TVNs may not be able to generate the force with the same amplitude as in feed-forward active control method. Therefore, Eq. (12) can now be written as

$$\mathbf{f}_{tr} = -\mathbf{K}_r[\boldsymbol{\Psi}^T\mathbf{q}_o] \tag{16}$$

where $\mathbf{K}_r$ is the required diagonal matrix of the dynamic stiffness of the TVNs that produce feedback forces with the same amplitude with its active control counterpart. This will enable us to express the required dynamic stiffness of the j-th neutralizer as

$$K_{jr} = -f_{jso}[\boldsymbol{\Phi}^T(x_j)\mathbf{q}_o]^{-1} \tag{17}$$

where f_{jso} is the j-th optimum secondary force and $\mathbf{q}_o$ is as given in Eq. (10). After some mathematical manipulations, the optimum tuning ratio of the j-th neutralizer can be written as

$$\alpha_{jo} = \sqrt{1 - \frac{M_j\omega^2}{\Re\{K_{jr}\}}} \tag{18}$$

where $\Re\{K_{jr}\}$ is the real part of the required dynamic stiffness of the neutralizer in Eq. (17). Substitution of the Eq. (18) into (13) gives the optimal dynamic stiffness of the neutralizer as

$$K_{jo} = -M_j\omega^2\left[\frac{1+i2\zeta_j\alpha_{jo}}{1-\alpha_{jo}^2+i2\zeta_j\alpha_{jo}}\right]. \tag{19}$$

Further substitution from Eqs. (19) and (10) into (12) gives the vector of the optimal feedback forces as

$$\mathbf{f}_{to} = -\mathbf{K}_o\boldsymbol{\Psi}^T\mathbf{q}_o. \tag{20}$$

This leads to the minimized kinetic energy of the host structure as

$$KE = \frac{M_s\omega^2}{4}\{\mathbf{f}_{to}^H\mathbf{G}^H\mathbf{G}\mathbf{f}_{to} + \mathbf{f}_{to}^H\mathbf{G}^H\mathbf{d} + \mathbf{d}^H\mathbf{G}\mathbf{f}_{to} + \mathbf{d}^H\mathbf{d}\}. \tag{21}$$

IV. SIMULATIONS RESULTS

Mathematically, it was shown in the previous section that there is a method to determine the optimal dynamic stiffness of the TVNs that is by using the quadratic minimization technique. The minimization technique was widely used in the feedforward active control method. In this section, it will be shown theoretically, through some computer simulations that the technique developed in this paper can be used to optimize the TVNs so that they can be as effective as active control method, for global control purpose at single frequency excitation.

To demonstrate the effectiveness of the optimized TVN, a simply supported beam is used as a host structure. The beam has the dimension of 1 m × 0.0381m × 0.00635 m, and the material density and Young's modulus are 7870 kg/m^3 and 207 GN/m^2 respectively. A physical force, operates at a single frequency at any time, is located at $x_f = 0.1L$ where L is the total length of the beam. An optimal neutralizer is fitted at $x_n = 9L/20$, which does not coincide with any nodal points in the frequency of interest, which is from 15 to 450 Hz.

The ratio between the mass of the TVN to the mass of the beam and the TVN damping ratio are fixed at 0.04 and 0.001 respectively ($\mu = 0.04$, $\zeta_t = 0.001$). These values were determined using the method described Brennan and Dayou [18]. The kinetic energy of the beam is shown in Fig. 1 for no control, for control with optimal TVN fitted at $x_n = 9L/20$ and for control with active control device located at $x_a = 9L/20$. It can be seen that the effectiveness of the control using the optimal TVN is similar to that of active control with the exception below 15 Hz. This is because as discussed above, the mass and the damping ratio of the neutralizer are optimized only at this frequency and above. At lower frequencies, the mass of the neutralizer has to be increased, which may involve an increase in the weight of the whole structure and for that reason, it become unsuitable and is not practical in real application.

The optimum value of tuning ratio of the neutralizer (refer to Eq. (18)) is shown in Fig. 2. It can be seen that in order to minimizes the kinetic energy stored in the system, the TVN is sometimes has to behave like a mass (when $\alpha_o < 1$) or stiffness like (when $\alpha_o > 1$). There are points however that the neutralizer has to simply tuned to the excitation frequency (when $\alpha_o = 1$) or better simply be removed (when the tuning ratio change from less then unity to more then unity) as discussed by Brennan and Dayou [18]. By choosing such value, it can be seen from Fig. 1 that tunable vibration neutralizer can be very effective and is comparable to that of active control method except at certain condition as discussed before.

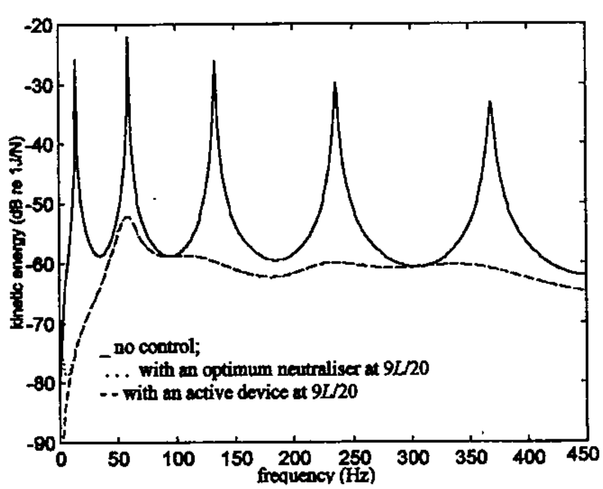

FIG. 1. Kinetic energy of the simply supported beam with no control, with optimum neutralizer attached at $9L/20$ and with active control device at $9L/20$ respectively. $\mu = 0.04$ and $\zeta_t = 0.001$.

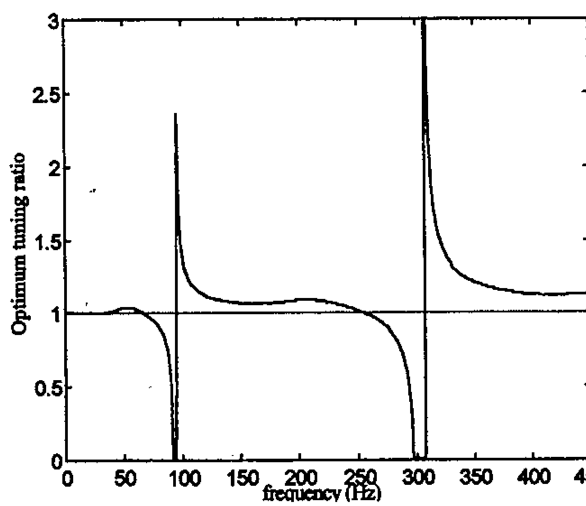

FIG. 2. Optimum tuning ratio, α_o of the tunable vibration neutralizer versus frequency of the excitation force.

V. CONCLUSIONS

A mathematical approach on how to use the quadratic minimization technique to optimize a tunable vibration neutralizer so that the global response of a structure is minimized has been discussed in this paper. The approach uses the quadratic minimization technique, which was widely used in active control method. Using this approach, the optimal tuning ratio of a tunable vibration neutralizer that minimizes the global response of a structure can be determined. This is theoretically proven through some computer simulations in this paper.

REFERENCES

[1] C. F. Beards, *Engineering Vibration Analysis with Application to Control Systems*, Edward Arnold, London (1995).

[2] J. Q. Sun, M. R. Jolly and M. A. Norris, "Passive, adaptive and active tuned vibration neutralizer - A survey", Transactions of ASME: Special 50th Anniversary Design Issue, **117**, 234-242 (1995).

[3] D. R. Thomas, The Active Control of the Transmission of Sound, Ph.D. Thesis, University of Southampton (1992).

[4] C. F. Ross and M. R. J. Purver, *Active Cabin Noise Control*, ACTIVE 97, Budapest, Hungary (1997).

[5] R. J. Bernhard, H. R. Hall and J. D. Jones, *Adaptive-Passive Noise Control*, Inter-noise, Toronto, Ontario, Canada (1992).

[6] C. Guigou, J. P. Maillard and C. R. Fuller, "Study of globally detuned absorbers for controlling aircraft interior noise", Fourth International Conference on Sound and Vibration, St. Petersburg, Russia (1996).

[7] M. A. Franchek, M. W. Ryan and R. J. Bernhard, "Adaptive passive vibration control", Journal of Sound and Vibration, **185**(5), 565-585 (1995).

[8] P. A. Nelson and S. J. Elliott, *Active Control of Sound*, Academic Press, London (1992).

[9] C. R. Fuller, S. J. Elliott and P. A. Nelson, *Active Control of Vibration*, Academic Press, London (1996).

[10] C. E. Crede, *Shock and Vibration Concepts in Engineering Design*, Prentice Hall, Englewood Cliffs, NJ (1965).

[11] M. J. Brennan, "Vibration control using a tunable vibration neutralizer", Journal of Mechanical Engineering Science (Part C), **211**,91-108 (1997).

[12] M. R. F. Kidner and M. J. Brennan, "Improving the performance of a vibration neutralizer by actively removing damping", Journal of Sound and Vibration, **221**(4),587-606 (1999).

[13] P. L. Walsh and J. S. Lumancusa, "A variable stiffness vibration neutralizer for the minimization of transient vibration", Journal of Sound and Vibration, **158**(2), 195-2111 (1992).

[14] H. von Flotow, A. Beard and D. Bailey, "Adaptive tuned vibration absorbers: Tuning laws, tracking ability, sizing and physical implementation", Proceedings of Noise Control, 437-454 (1994).

[15] M. J. Brennan, "Actuators for active vibration control - tunable resonant devices", Proceedings of the 4th European Conference on Smart Materials and Structures, Harrogate, 41-48 (1998).

[16] M. J. Brennan and J. Dayou, "A comparison between active and semi-active global vibration control of structures", Joint 137th Meeting of the Acoustical Society of America and 2nd convention of the European Acoustics Association, Technical University of Berlin, Germany (1999).

[17] D. I. G. Jones, "Response and damping of a simple beam with tuned damper, Journal of Acoustical Society of America, **42**(1), 50-53 (1967).

[18] M. J. Brennan and J. Dayou, "Global control of vibration using a tunable vibration neutralizer", Journal of Sound and Vibration, **232**(3), 585-600 (2000).