Increasing the bandwidth of the width-split piezoelectric energy harvester

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A N T I C I P L E A C E N T

A new method to maximize the output power of a piezoelectric energy harvesting system has been previously proposed by the authors. This can be achieved by reducing the mechanical damping through folding a given piezoelectric material equally and splitting it into smaller width. Experimental results have shown that the power harvested increases when the number of fold increases but with the trade off the optimal operating frequency range, which is referred as the bandwidth. This paper aims to improve the bandwidth by modifying the natural frequency of each split piezoelectric material and connecting them in parallel. Experimental results show that the bandwidth increases as the difference between the natural frequency of the reduced-width piezoelectric materials increases. Although these results are with trade off in reducing output power gain, the gain in the bandwidth per unit output power reduction is still increasing. This shows that the maximum output power of the harvesting system can be ensured with the width-splitting method and the bandwidth of the output can be widened by increasing the difference between the natural frequencies of the participating piezoelectric elements. This maximization method with wideband feature can be implemented at microscopic stage to be incorporated in the microelectronics devices such as MEMS.

1. Introduction

The concept of harvesting environmental energy to power up low power consuming devices, such as MEMS and wireless sensing nodes, is getting popular to be adopted in the manufacturing design of the microelectronics technology. The electromagnetic energy harvester can generate high power using an induced magnetic field in the device schemes [1]. However, generating a magnetic field with MEMS is not easy because of miniaturization. The electrostatic method has an advantage in it’s device size, but it requires a high frequency and external voltage sources [2]. On the other hand, the piezoelectric energy harvester is relatively easy to fabricate by MEMS technique and can be designed to obtain the target frequency with high conversion efficiency because it provides the simplest setup and flexibility in dimension control [3–8]. Furthermore, the energy harvesting system using piezoelectric materials is commonly in the form of a cantilever beam embedded to the system [9–16]. Hence, it’s structural setup is simple and also provides the flexibility to alter it’s physical dimensions as well as it’s equivalent mass for the best harvesting performance.

In the study of the mechanical vibration on the cantilever beam, it can be realized that the transmissibility of vibration energy from the driving system to the beam relies on the damping level at the vibrating beam and the matching level of the input frequency to the beam natural frequency. Higher damping causes the dissipation of input energy and hence reduces the transmissibility. Frequency mismatch of a few percent results in a significant drop in vibrating displacement of the beam and this refers to narrow bandwidth. In order to maximize upon the existing optimization techniques for further improving the energy harvesting performance over a wider range of input frequencies with the specific dimensions of piezoelectric materials, the considerations in lowering the damping level and widening the bandwidth are essential for any latest MEMS design based on the piezoelectric harvester.

A method has been proposed by the authors to increase the energy harvesting by folding equally and then splitting a given dimension of piezoelectric material with the pre-defined dimensions so that the conversion of energy is more efficient by reducing the mechanical damping of the piezoelectric materials during the vibration [17]. This method is, therefore, referred as the width-splitting method. It can be incorporated well with other optimization techniques for maximizing the output power. The experimental results of this work show that the power harvested has increased 2.45 times by the first folding and 2.76 times by the second folding as compared to without folding. The results also show that the output power is optimum only when the driving frequency matches the beam natural frequency and the reducing damping is with the trade-off of the narrow
bandwidth. Since the ambient vibrations are normally random and wideband in nature, and they consequently do not always match the beam natural frequency, the output power is not always harvested to be the maximum with the width-splitting method. In order to extract more energy from such vibrations, the harvesting system should have sufficient bandwidth to cover peak power frequencies of the input vibrations. This paper is an extension of the previous work aimed at increasing the bandwidth of the width-split piezoelectric energy harvester by altering the natural frequency of each participating split beam and connecting together in parallel.

2. Theoretical derivation of the width-splitting method

The width-splitting method for maximizing the piezoelectric energy harvesting using cantilever beam has been described previously [17]. However, the theoretical formulation is summarized here for convenience.

The composite cantilever beam used as harvester described in the previous work consists of a layer of piezoelectric material and a layer of substrate as a host structure. They are selected to have the same length, \( L \), and the same width, \( w_0 \), so that one layer is completely adhered to another. This piezo-substrate composite beam, herewith known as piezoelectric beam, is strongly clamped at one end with free condition at the other end. This makes the harvester behave like a cantilever beam. External stress to the piezoelectric material can be induced by fitting the clamped end on a vibration source that causes deflection and bending.

During the base vibration, the external stress causes deflection and bending to the piezoelectric beam. The deflection distorts the internal dipole moments within the piezoelectric material and produces electrical field strength along the \( z \) direction given by

\[
E_z = -g_{31} \sigma_{\text{piezo}} + \frac{D_z}{\varepsilon_{r} \varepsilon_0},
\]

where

\[
\sigma_{\text{piezo}} = Y_p (e_{\text{piezo}} - g_{31} D_z),
\]

\( e_{\text{piezo}} \), \( g_{31} \), \( D_z \), \( \varepsilon_r \), and \( \varepsilon_0 \) are the strain on the piezoelectric material, the piezo stress constant and the electrical displacement along the \( z \) direction, the permittivity of the vacuum and the relative permittivity of the material, respectively.

When the piezoelectric beam is used to harvest electric energy, it can be modeled as an equivalent circuit which consists of a series of piezoelectric capacitance, \( C \), with a voltage source [20] (refer to Fig. 1).

The electric charge harvested on the beam and the piezoelectric capacitance derived from the previous work can be expressed, respectively, as

\[
Q = -3AB(1 - A + AB)g_{31} e_r \varepsilon_0 L^2 F
\]

\[
C = \frac{Q}{V} = \frac{(1 - A + AB)jh_0 L^2 \varepsilon_0}{(1 - A)h_0 k},
\]

where

\[
A = \frac{L_c}{L_0},
\]

\[
B = \frac{Y_s}{Y_p},
\]

\[
t_b = t_s + t_c.
\]

During the base vibration, the force experienced by the beam, \( F \), can be written as

\[
F = Kd_0 \sqrt{(2\xi)^2 + \frac{1}{(2\xi)^2 + r^2}} \sin(2\pi ft),
\]

where \( \xi = \frac{\Delta f}{2f_0} \),

\[
\xi = \frac{\Delta f}{2f_0},
\]

\( \Delta f \) is the 3 dB bandwidth which is referred as the range of frequencies for the output power is at half of the maximum power at \( f_0 \).

Consider a given piezoelectric beam with initial width of \( w_0 \) and length \( L \). This beam is folded and then split equally into two identical beams with smaller width. Since this is the first folding, this is known as 1-fold (i.e. \( N=1 \)) of the beam. Further equal-splitting of each of the two identical beams produces four identical beams, and is referred as 2-fold (\( N=2 \)) and so on. 0-fold (\( N=0 \)) refers to the initial beam before any splitting, and the folding is illustrated in Fig. 2. It can be seen that \( N \) number of folding produces \( 2^N \) number of identical beams.

When the piezoelectric beam is split, each of the reduced width beams produces the same electrical charge output as the initial beam (with the width of \( w_0 \)) when they are set into vibration (refer to Eq. (3)). And, by connecting all the split beams in parallel configuration, and since all beams are identical, the
total electric charge harvested from the beams add up together which can be expressed mathematically as

$$Q_N = -3ABQ^N \frac{(1-A)(1+AB)g_{33}L_0^2}{t_b}.$$  

(12)

It can be seen that overall, the electrical charges increases as the number of splits increase. However, the effective piezoelectric capacitance remain the same regardless of how many folds the beam is split which is

$$C_N = \frac{(1-A)(1+AB)h_0L_0}{t_b}.$$  

(13)

The open circuit voltage of the piezoelectric beam can be obtained by using the formula $V_O = Q_N/C_N$ with the substitution of Eqs. (12) and (13) and then be expressed as

$$V_N = -3ABQ^N \frac{(1-A)g_{33}L_0^2}{h_0t_b}.$$  

(14)

The dynamic model of the piezoelectric beam can be derived by substituting Eq. (10) to Eq. (14) as

$$V_N = -3ABQ^N \frac{(1-A)g_{33}L_0K_0}{h_0t_b} \sqrt{\frac{2(2\pi^2f^2 + 1)}{(2\pi^2f^2 + 1 - r^2)} \sin(2nf_t)},$$  

(15)

where $\zeta_N$ is the damping ratio of the $N$-fold beams. It should be noted here that as the number of folding $N$ increase, the width of each beam become small. Therefore, the internal damping of the beam further reduces. The new damping ratio of the $N$-fold beam can be determined experimentally using equation

$$\zeta_N = \frac{\Delta f_N}{2f_{31}},$$  

(16)

where $\Delta f_N$ and $f_{31}$ are the 3 dB bandwidth of the $N$-th fold beams and the natural frequency of the beams, respectively.

When an external resistive load is connected in parallel to the $N$-fold beams under harmonic excitation (refer Fig. 3), the load voltage can obtained using the concept of potential divider as

$$V_{load,N}(t) = \frac{R_{load}V_N}{\sqrt{R_{load}^2 + (2\pi f C_N)^2}}.$$  

(17)

The root-mean-square (rms) voltage across the resistive load can be obtained by first substituting Eq. (15) into Eq. (17) which gives after rearrangement as

$$V_{load,N, rms} = \left(-3ABQ^N \frac{(1-A)g_{33}L_0K_0R_{load}}{h_0t_b} \right) \times \left( \frac{2(2\pi^2f^2 + 1)}{2(R_{load}^2 + (2\pi f C_N)^2)} \right)^{1/2}.$$  

(18)

The average power output to the resistive load from the split piezoelectric beams in $N$-fold can therefore be obtained using the well known formula given by

$$P_N = \frac{V_{load,N, rms}^2}{R_{load}}.$$  

(19)

3. Wide bandwidth energy harvester

As discussed in the previous work [17], the output power increases as the damping on the piezoelectric beams reduces through width-splitting method but with the trade-off of the narrow bandwidth. The works on wideband operations in energy harvesting have been described in many articles for example [22–28]. Among the various proposed methods, the most suitable approach for applying to the piezoelectric beam described in this paper is to modify the natural frequency of each participating beams.

The natural frequency of the piezoelectric beam can be determined by using

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K}{M}}.$$  

(20)

Following the work by Ref. [29], the equivalent dynamic stiffness, $K$, of the beam described in this paper can be expressed as

$$K = \frac{Y_0W_0L_0^3h}{4(1-A)(1+AB)l^4}.$$  

(21)

The equivalent mass, $M$, of the beam with end mass attached at free end as illustrated in Fig. 4 is given by

$$M = \frac{33}{140}M_{beam} + M_{end}.$$  

(22)

where $M_{beam}$ and $M_{end}$ are the masses of the piezoelectric beam and the end mass, respectively.

Researchers from University of Oulu have proposed to vary the length of each beam to change the bandwidth of the harvesting system [28]. Since their work focuses only on the wideband operation, the power density harvested reduces because they use a large volume and weight of the non-resonant beams for widening the bandwidth. In order to secure the high output power as well as wide bandwidth of the specific size of the piezoelectric harvester discussed in this paper, the natural frequency for each reduced-width beams described in Section 2 needs to be altered so that they are all having different natural frequencies before they are connected in parallel. Since the initial dimensions of the piezoelectric beam are given in this paper, the proposal in varying...
the length of each reduced-width beam is not applicable here. Referring to Eqs. (20) and (22), noting that

$$f_0 \propto \frac{1}{\sqrt{33/140}} \frac{M_\text{beam} + M_\text{end}}{M_\text{beam}}.$$  

(23)

The alteration of the natural frequency of each of the reduced-width beams in this paper can be achieved by modifying the end mass attached to each beam as shown in Fig. 4.

4. Experimental results and discussion

The experiment was carried out with the setup shown in Fig. 5. The experiment started with the initial piezoelectric beam and then continued with splitting the initial piezoelectric beam equally into two (1-fold) and also into four (2-fold), respectively, under the same setup and same experimental procedures. The piezoelectric beams in 0-fold, 1-fold and 2-fold are shown in Fig. 6(a)–(c), respectively.

The detail of the experiment for verifying the width-splitting method as described in Ref. [17] is briefly described here for convenience of readers. The composite cantilever beam of a thin piezoelectric material bonded on a thin flexible polypropylene, acts as a host structure, was fabricated for the experiment. The piezoelectric material was a polymer base manufactured by Measurement Specialties, Inc. (MSI). It had 52 µm thickness with initial length and width of 45 mm and 20 mm, and its Young’s modulus of $3 \times 10^9$ N m$^{-2}$. The host material was made of polypropylene with thickness of 124 µm and Young’s modulus of $9.9 \times 10^9$ N m$^{-2}$. The piezo stress constant, $g_{31}$, and the relative permittivity, $\varepsilon_r$, were given as $216 \times 10^{-8}$ m$^2$ C$^{-1}$ and 13, respectively, and the effective mass of the initial beams was 0.15 g. With this configuration, theoretically, the single piezo beam had its natural frequency of 27.2 Hz. Therefore, the piezo beam was excited using harmonic base displacement with excitation frequency, $f$, in the range of 21–32 Hz and the base displacement, $d_b$, was set to 2.0 mm. Throughout the experiment, a shaker powered by frequency generator was used to provide base vibration to the piezo beam. A resistor of 2 MΩ was selected as the load. The load voltage was measured and calculated for each of the set vibration frequency.

In order to obtain the maximum output power from the piezoelectric harvester, the external load must have the impedance matching with the internal impedance of the piezoelectric material. From the equivalent circuit of the piezoelectric harvester as shown in Fig. 4, internal impedance is found to be the capacitive reactance of piezoelectric capacitance, $1/2\pi fC_0$. Substitute Eq. (18) into Eq. (19) and then differentiate the resultant equation with respect to the resistive load (i.e. $dP_N/dR_{\text{load}}$), the optimum load can be identified theoretically by taking $dP_N/dR_{\text{load}} = 0$ as

$$R_{\text{load}} = \frac{1}{2\pi fC_N}.$$  

(24)

The actual impedance matching for beam in 0-fold is approximately 2.8 MΩ and that in 2-fold is approximately 1.7 MΩ. A load resistance of 2 MΩ, which is close to these matching values, is chosen in the experiment for the convenience of comparing the harvesting performance to a fixed resistance load.

Considering the load effect on the oscilloscope during voltage measurement, the actual resistance of the load has to be adjusted so that the effective load resistance is 2 MΩ exactly. The input impedance of the oscilloscope chosen for the measurement is 10 MΩ and the actual resistance of the load used in experiment is 2.5 MΩ. These produce the effective resistance of 2 MΩ when the oscilloscope is connected to this resistance in parallel.

Fig. 7 shows the comparison of simulation and experimental results described in Ref. [17]. $N$ is the number of splitting of the initial given piezoelectric material. Detailed inspections of Fig. 7 show that there could be certain number of splits that beyond this will give no further significant increment in the electrical power output. However, this is beyond the scope of the present paper. The main objective of the paper is to provide evidence that splitting the piezoelectric harvester will increase the harvested power output.

From the classical beam theory, the resonant frequency of the beam is independent on it’s width but is dependent on it’s thickness and length. However, damping effect is decreasing according to the decrease of the surface area of the beam in vibrating. With the constant thickness and length of the beam, when the width of the beam decreases, the surface area reduces and, hence, the damping degree of the vibration decreases. Under the damped vibration, the largest vibrating amplitude, which results in highest power harvesting, occurs at the damped resonant frequency. From the damping theory, the damped resonant frequency is always lower than the undamped resonant frequency. It is understood that the variation between the damped resonant frequency and undamped resonant frequency decreases as the damping degree decreases. In Fig. 7, the beam in 0-fold has the larger surface area as compared to that of the beam in 1-fold. Therefore, damped resonant frequency for the beam in 0-fold deviate more below the undamped resonant frequency than that in 1-fold deviate less below the undamped resonant frequency as beam in 0-fold suffers greater damping than the beam in 1-fold.

Table 1 summarizes the result of the experiment from the previous work. It can be seen that the bandwidth decreases as the number of folding increase. As a consequence, the damping ratio of the harvester also reduces. This is corresponds to a shaper peak in the power responds of the harvester as shown in Fig. 4. It can also be seen that due to the reduction in damping value, there is substantial increment in the power output when the given piezoelectric film is folded and split. For example, for single folding ($N=1$), the power output increased to 47 µW compared to 19.2 µW without folding. This is corresponding to 145%. Further increase is observed for two folding, which is as high as 176%.
Based on the experimental results, increasing number of fold causes decreasing damping ratio. Therefore, an equation is proposed to relate the damping ratio of the N-fold to 0-fold as

$$\frac{\zeta_0}{\zeta_N} = 2^N,$$

where $\zeta_0$ is the damping ratio of the energy harvesting system when the piezoelectric beam is in 0-fold. This proposed relation is investigated by plotting a graph of $\log \zeta_N$ against $N$ which is shown in Fig. 8 and $x$ is found to be 1.227.

From Fig. 7, it is noted that the bandwidth is getting narrower when the folding number increases. In order to see the general trend with higher folding number, simulations of the output powers at the 2 MΩ optimum load for piezoelectric beam in 3-folding, 5-folding and 7-folding are performed by using Eqs. (19) and (25). The outcome of the simulations, as illustrated in Fig. 9, shows that the bandwidth becomes smaller as $N$ increases. Therefore, there is a need to increase the bandwidth. The experiment results and the simulations, as shown in Figs. 7 and 9, respectively, are absolutely without the consideration of using the system for wideband operation. Following the concept described in Section 3, the end masses with each of them attached to each reduced-width beam can be adjusted so that there is a variation of natural frequency among the split beams. These beams are then connected in parallel for increasing the bandwidth, $Df_{3dB}$, of the piezoelectric energy harvester.

The variation of natural frequency of 1, 2 and 3 Hz are therefore chosen for the following experiment. For 1-fold, there are only two split beams. If the variation is set at 1 Hz, one beam remains its original natural frequency (i.e. 27.2 Hz as described in the previous paper) and another is adjusted to 26.2 Hz by modifying the end mass. For 2-fold, there are four split beams in total. If the variation is set at 1 Hz, one beam remains its original natural frequency and other three are adjusted with equal interval to 26.9, 26.5 and 26.2 Hz, respectively. The similar experimental procedures are followed for the variation of 2 and 3 Hz to the split beams in 1-fold and 2-fold. These beams are excited using harmonic base displacement with excitation frequency, $f$, in the range of 21–32 Hz and the base displacement, $d_0$, is set to 2.0 mm. The voltage drop at the resistor is measured using a volt meter at different excitation frequency, and later the power is calculated using the standard formula given in Eq. (19). The experimental results are shown in Figs. 10 and 11 for piezoelectric beam in 1-fold and 2-fold, respectively.

Table 2 summarizes the experimental results for bandwidth study. It can be seen that the bandwidth increases as the variation of natural frequency increases. This is corresponds to a lower peak in the power responds of the harvester as shown in Figs. 10 and 11. However, the gain in the bandwidth per unit output power reduction increases with increasing variation of natural frequency. This shows that the maximum output power of the harvesting system is still ensured with wider bandwidth feature.

5. Conclusion

In this paper, the method of modifying the natural frequency of each participating beams to increase the bandwidth of the

Table 1

<table>
<thead>
<tr>
<th>Folding number (N)</th>
<th>Number of identical piezo and the corresponding width (mm)</th>
<th>Bandwidth (f/Hz)</th>
<th>Damping ratio, $\zeta_N$ (Eq. (17))</th>
<th>Output power to the external load of 2 MΩ ($\mu$W)</th>
<th>Power increase compared to N=0</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=0</td>
<td>Single piezo, 20 mm</td>
<td>6.5 Hz</td>
<td>0.119</td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>N=1</td>
<td>Two piezo, 20 mm</td>
<td>5.3 Hz</td>
<td>0.097</td>
<td>47.0</td>
<td>145%</td>
</tr>
<tr>
<td>N=2</td>
<td>Four piezo, 5 mm</td>
<td>4.3 Hz</td>
<td>0.079</td>
<td>53.0</td>
<td>176%</td>
</tr>
</tbody>
</table>
Fig. 8. Relationship between log $\xi_n$ and the number of fold, $N$.

Fig. 9. Simulations of power harvesting for the optimum load with piezoelectric beam in 3-fold, 5-fold and 7-fold, respectively.

Fig. 10. Comparison of load power for piezoelectric beam in 1-fold with no variation in natural frequency, variations of 1 Hz, 2 Hz and 3 Hz, respectively.
piezoelectric energy harvester has been explored theoretically and experimentally. The experiment results show that the bandwidth increases when the variation of the natural frequency increases although there is reduction on the overall gain in output power. The output power of the piezoelectric energy harvesting system can be maximized with the width-splitting method in higher folding number and the bandwidth of the output can be increased by wider variation in natural frequency.

Acknowledgments

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References


Table 2
Summary of experimental results for bandwidth study.

<table>
<thead>
<tr>
<th>Folding number</th>
<th>Variation of natural frequency (Hz)</th>
<th>Natural frequency of each reduced-width beam (Hz)</th>
<th>$P_{load}$ (nW)</th>
<th>Reduction (%)</th>
<th>$f_{BW}$ (Hz)</th>
<th>Gain (%)</th>
<th>$\Delta f_{BW}/P_{load}$ (J^-1)</th>
</tr>
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<tbody>
<tr>
<td>$N=0$</td>
<td></td>
<td>27.2</td>
<td>19.2</td>
<td>6.5</td>
<td>6.4</td>
<td>7.0</td>
<td>32.0</td>
</tr>
<tr>
<td>$N=1$</td>
<td></td>
<td>27.2</td>
<td>47.0</td>
<td>5.3</td>
<td>6.4</td>
<td>7.0</td>
<td>32.0</td>
</tr>
<tr>
<td>1</td>
<td>27.2; 26.2</td>
<td>44.0</td>
<td>12.8</td>
<td>5.8</td>
<td>7.0</td>
<td>32.0</td>
<td>283</td>
</tr>
<tr>
<td>2</td>
<td>27.2; 25.2</td>
<td>39.0</td>
<td>17.0</td>
<td>5.8</td>
<td>7.0</td>
<td>32.0</td>
<td>283</td>
</tr>
<tr>
<td>$N=2$</td>
<td></td>
<td>27.2</td>
<td>53.0</td>
<td>4.3</td>
<td>7.0</td>
<td>32.0</td>
<td>283</td>
</tr>
<tr>
<td>1</td>
<td>27.2; 26.9; 26.5; 26.2</td>
<td>49.0</td>
<td>7.5</td>
<td>5.0</td>
<td>7.0</td>
<td>32.0</td>
<td>283</td>
</tr>
<tr>
<td>2</td>
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<td>44.0</td>
<td>17.0</td>
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$P_{load}$ is the output power for the 2 MΩ resistive load; $\Delta f_{BW}/P_{load}$ is the gain in the bandwidth per unit output power reduction.


