Global control of vibration using a tunable vibration neutraliser

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Tunable vibration neutralisers are currently used to control harmonic vibration of structures at the point at which they are attached, and the way in which the characteristics of the neutralisers influence their effectiveness in this role are reasonably well understood. However, the use of tunable vibration neutralisers to control global vibration of a structure has not been so well explored, and in this article the neutraliser parameters that influence the vibrational kinetic energy of a host structure are highlighted. It is shown that in general a de-tuned rather than a tuned vibration neutraliser is required at most frequencies, offering a stiffness-like or a mass-like impedance to the structure. However, because this control strategy requires a measure of the global vibration of the host structure, which can be difficult and/or costly in practice, it is preferable to use a tuned vibration neutraliser. For this to be effective it is demonstrated that the tuned neutraliser needs to be appropriately positioned on the structure. If this is the case then such a device can be almost as effective as a de-tuned neutraliser over quite a wide range of frequencies.

1. Introduction

Since the vibration absorber was described by Ormodroyd and den Hartog in 1928 [1], it has been used in many applications [2]. Although many researchers and engineers still call it the vibration absorber, it has been called a vibration neutraliser by others, for example [2,3], and it is referred to by this name in this article. A fairly recent development has been to make the device adaptive, by making the stiffness adjustable, so that it can track changes in the excitation frequency [4-7], and ways of achieving this have been reviewed by Von Flotow *et al* [5] and Brennan [8]. One application of this device that is worthy of mention is the control of transmitted vibration from the engine of a DC9 aircraft to the fuselage and hence a reduction in sound pressure inside the cabin [9]. These types of systems, called adaptive-passive by Franchek *et al* [10], are an attractive alternative to passive vibration control. They offer an improved performance over passive measures, and offer a solution, which although does not match that of active control, can potentially work with a very simple control system, such as that described by Long *et al* [11]. Additionally, because the devices are used at resonance, large forces can be generated, and the only power required is to tune the system, which can be very small. This has obvious benefits with large structures [12].

Recently, researchers have turned their attention to the use of vibration neutralisers to control sound transmission through structures, for example [13,14], with the particular application of controlling sound transmission into aircraft in mind. It was found that it was preferable to use *de-tuned* vibration neutralisers rather than resonant devices to reduce the global sound pressure level inside the cabin [15]. To achieve this in practice requires a number of sensors to measure the global vibration or sound pressure, and then the use of a vibration neutraliser becomes less attractive. In this article, the conditions under which a *tuned* vibration neutraliser would be appropriate to control the global vibration of the structure are highlighted. If the right conditions can be arranged then a simple control strategy can be used [11], and this makes the use of such a resonant device an attractive alternative to fully active control. A beam is chosen as the host structure to illustrate the principle, but the conclusions can be generalised to any structure which has well-spaced resonance frequencies.

2.Global control using a tunable vibration neutraliser

The time averaged kinetic energy E, of the structure is taken as the measure of global vibration of the structure and is proportional to the square of the modal velocity amplitudes. For a beam it is given by [16]:

$$E = \frac{m\omega^2}{4}\mathbf{q}^{\mathrm{H}}\mathbf{q} \qquad (1)$$

where m is the mass of the beam, the superscript $^{\mathrm{H}}$ denotes the Hermitian transpose and q is the vector of modal amplitudes given by [17]:

$$\mathbf{q} = \mathbf{A}\left[\varphi(x_f) - \varphi(x_n)\left(\frac{k_n\alpha_{nf}}{1 + k_n\alpha_{nn}}\right)\right]f \qquad (2)$$

where α_{nn} is the point receptance of the structure at the point where the neutraliser is attached and α_{nf} is the transfer receptance between the primary force and the neutraliser position, and $\mathbf{A}$ is a diagonal matrix of modal amplitudes. $\varphi(x_f)$ and $\varphi(x_n)$ are the vectors of mode shapes evaluated at the primary force and neutraliser positions respectively. k_n is the dynamic stiffness of the neutraliser given by:

$$k_n = \frac{-\omega^2 m_n\left(1 + j2\zeta_n\frac{\omega}{\omega_n}\right)}{1 - \frac{\omega^2}{\omega_n^2} + j2\zeta_n\frac{\omega}{\omega_n}} \qquad (3)$$

where m_n, ζ_n and ω_n are the mass, damping ratio and circular natural frequency of the neutraliser respectively.

To examine the differences between global and local control, the cantilever beam shown in figure 1a is used as an example structure. Because the vibration neutraliser is tunable, the spring stiffness is adjustable as shown in the figure. If an aluminium cantilever beam of dimensions 0.5 m × 0.05 mm × 0.0055 mm, is excited at x/L=0.2, and the neutraliser is tuned to a fixed frequency of 125 Hz, then the kinetic energy of the beam with and without the neutraliser attached is shown in figure 2. The characteristic dip (marked A) in the kinetic energy can be seen at 125 Hz with a peak either side of the dip. If the dip in the kinetic energy of the beam is calculated at *each frequency*, with the neutraliser tuned so that it is *resonant at each frequency*, then the resulting plot is shown in figure 3, with the kinetic energy of the beam alone for comparison. It can be seen from this graph that although there is a reduction in the kinetic energy at some

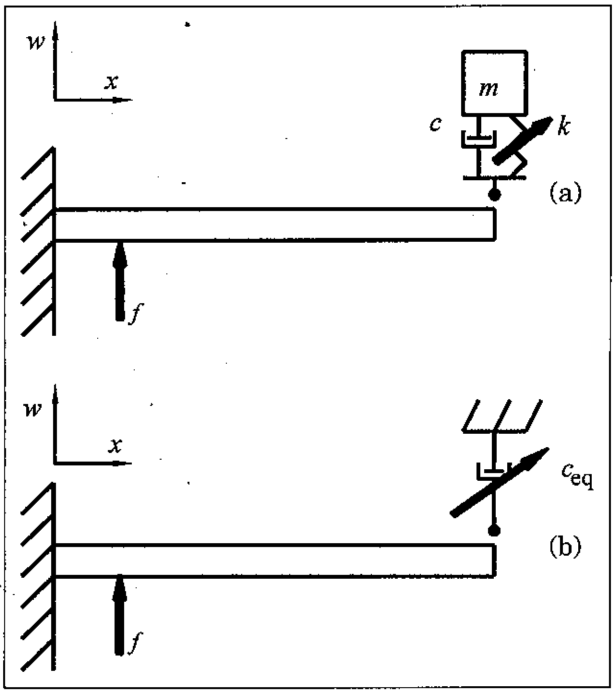

Figure 1. Cantilever beam with (a) a neutraliser attached at the free end, and (b) an equivalent damper attached

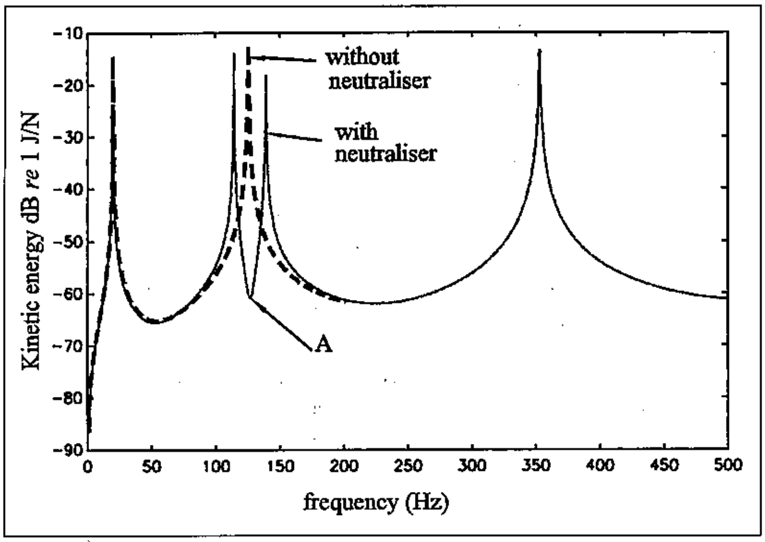

Figure 2. Kinetic energy of a cantilever beam with a neutraliser turned to 125 Hz and attached at the free end of the beam. Point A denotes the kinetic energy of the beam at the tuned frequency of the neutraliser

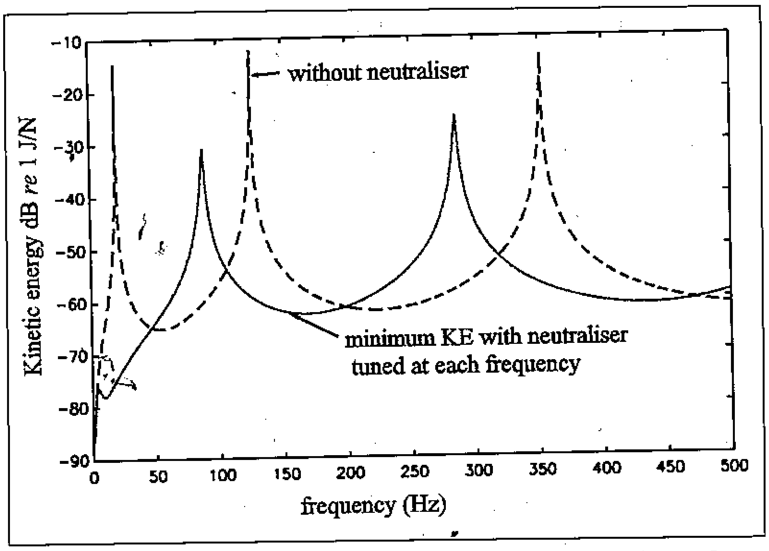

Figure 3. Kinetic energy of a cantilever beam without the neutraliser attached and with the neutraliser fitted and tuned to be resonant at each frequency

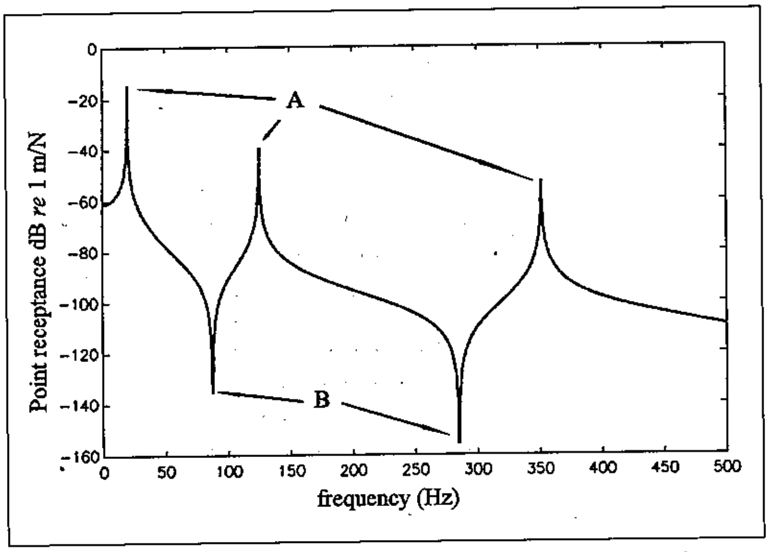

Figure 4. Point receptance of the cantilever beam when excited at the free-end. A – natural frequencies of the beam without the neutraliser attached; B – natural frequencies of the beam pinned at the neutraliser position (free end)

frequencies, then at other frequencies the tuned neutraliser actually causes an increase in global vibration. This has been observed by Fuller *et al* [15] in the control of sound transmission through structures, which is also a global control problem. The reason for this can be seen by examining the tuned dynamic stiffness of the neutraliser. Setting $\omega=\omega_n$ and assuming that damping is small, i.e., $\zeta_n \ll 1$, then from equation (3), the dynamic stiffness at the tuned frequency is given by:

$$k_{n(\text{tuned})} = \frac{j\omega_n^2 m_n}{2\zeta_n} \qquad (4)$$

At each frequency, this is equivalent to attaching a damper which is grounded at one end to the beam as shown in figure 1b. The equivalent damping coefficient is given by:

$$c_{eq} = \frac{\omega_n m_n}{2\zeta_n} \qquad (5)$$

It can be seen that this equivalent damping coefficient increases with mass and frequency and decreases with damping ratio. At frequencies when the effect of the tuned vibration neutraliser is most detrimental (about 85 Hz and 290 Hz in figure 3) the dynamic stiffness of the damper is much greater than the dynamic stiffness of the beam at the neutraliser attachment point, and this results in a pinning of the beam at this frequency. The resulting structure has a natural frequency at this frequency and hence the tuned vibration absorber has no beneficial effect. Indeed its effect is to create a structure that has a natural frequency which coincides with the forcing frequency. In this case it is better to de-tune the neutraliser as suggested by Fuller *et al* [15], so that it appears mass or stiffness-like. The problem with a de-tuning control strategy, however, is that the kinetic energy of the host structure needs to be measured, which entails distributed measurements over the whole structure to obtain a measure of the global vibration. It would be preferable to tune the neutraliser so that it was resonant because this means that a simple control algorithm could be employed that uses only acceleration signals at the base of the neutraliser and on the neutraliser mass, as discussed by Long *et al* [11].

To investigate the parameters that govern the large response when the neutraliser is tuned the denominator in equation (2) is set to zero, i.e.,

$$1 + k_{n(\text{tuned})}\alpha_{nn} = 0 \qquad (6)$$

At the frequency of interest $k_{n(\text{tuned})} \gg 0$, which means that $\alpha_{nn} \to 0$. Thus the anti-resonance frequencies of the beam at the position where the neutraliser is attached, indicate the frequencies at which a tuned neutraliser cannot effect global control of the structure. Figure 4 shows the free-end point receptance of the beam pictured in figure 1a without the neutraliser attached. The peaks in this frequency response function are the natural frequencies of the beam without the neutraliser attached, and the antiresonances indicate the problematic frequencies for that neutraliser position. It is known that anti-resonance frequencies are a function of position on the structure [18], and hence the frequencies at which global control cannot be achieved with a tuned neutraliser are dependent upon the position of the neutraliser.

The above analysis shows that provided the neutraliser is positioned correctly on the structure, it can be tuned to be a resonant device and hence global control of the structure can be achieved using a locally controlled tunable neutraliser. The question as to whether this control is optimal is addressed in the next section, but it is clear that provided some simple measurements are taken before fitting a tunable vibration neutraliser to ensure correct placement of the device, then a simple control system can potentially be used.

The effect of changing the position of the neutraliser can be seen in figure 5. This figure shows the way in which the kinetic energy of the third mode of a cantilever beam, excited at x/L=0.2 by a point force, changes as a function of neutraliser position and the ratio of mass of the neutraliser to the beam, μ. It is clear that the reduction in kinetic energy is a function of both mass ratio and position. Some general observations are

- a neutraliser placed at a nodal point on the host structure has no effect
- a neutraliser generally becomes more effective as it is placed closer to the source
- at some positions there appears to be a threshold mass ratio, beyond which there is no improvement in performance

To investigate these effects equation (2) can be examined under certain conditions. If the neutraliser is tuned to be resonant so that $|kn| \gg 1$ and the frequency of interest is not close to an antir-resonance of the point receptance α_{nn} then

$$|k_n \alpha_{nn}| \gg 1$$

and equation (2) becomes:

$$\mathbf{q}_{\mathbf{min}} = \mathbf{A}\left[\varphi(x_f) - \varphi(x_n)\frac{\alpha_{nf}}{\alpha_{nn}}\right]f \qquad (8)$$

which can be substituted into equation (1) to determine the minimum kinetic energy of the structure. Inspection of equation (8) shows that provided the condition given in equation (7) holds then surprisingly the kinetic energy of the host structure is independent of the dynamic stiffness of the tuned neutraliser. If the neutraliser is placed at the excitation position then the subscript f becomes n and $\mathbf{q}_{\mathbf{min}}$=0, which means that the kinetic energy of the beam can, in principle, be set to zero. It is possible to determine an expression for the optimum dynamic stiffness of the neutraliser provided attention is restricted to a frequency close to the *m-th* natural frequency of the original structure. In this case the response is governed by the amplitude of the *m-th* mode which can be normalised to the modal amplitude of the structure without the neutraliser attached to give:

$$q_{m(\text{norm})} = \frac{1}{1 + \dfrac{\mu\varphi^2_m(x_n)}{4\zeta_m\zeta_n}} \qquad (9)$$

where ζ_m is the damping ratio of the *m-th* mode of the beam. It is clear that the amplitude of this mode reduces as the mass ratio increases, and the damping in the neutraliser and the structure decrease. The positional dependence is also apparent; the neutraliser is most effective if it positioned on an antinode and is ineffective if it is placed on a node.

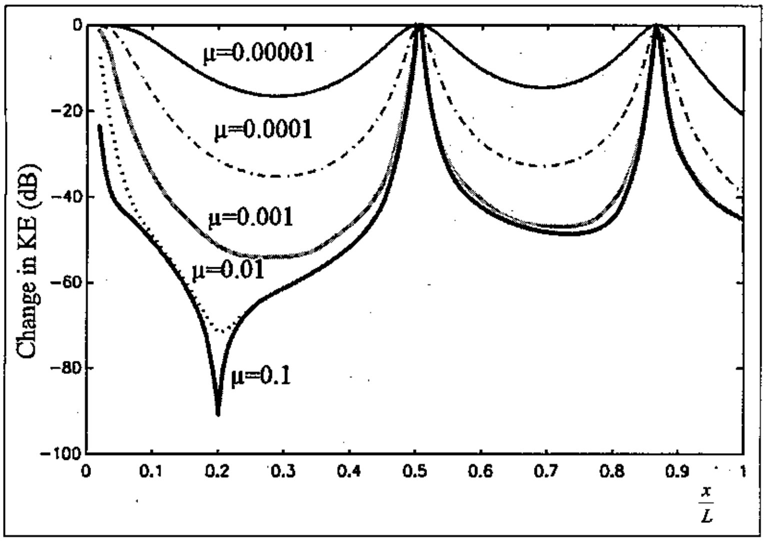

Figure 5. Change in kinetic energy of the beam as a function of non-dimensional position of the neutraliser on the cantilever beam. The beam is excited at its third natural frequency

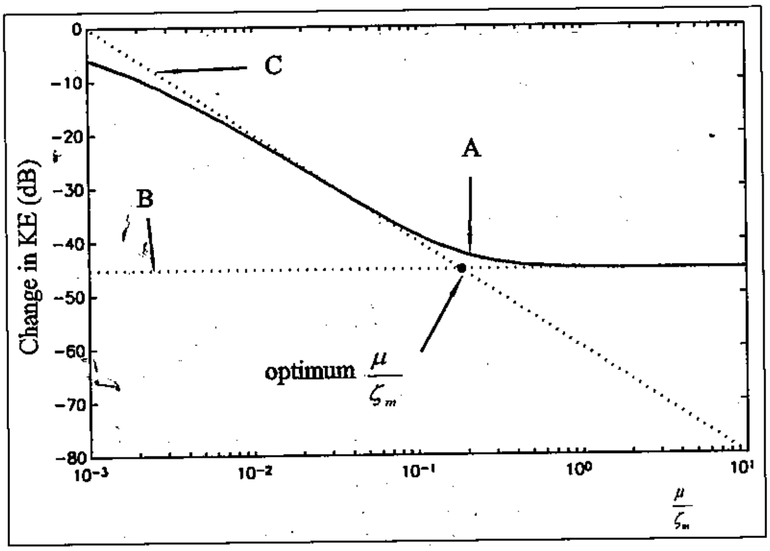

Figure 6. Change in the kinetic energy of the beam as a function of mass and damping ratios. A - actual change in the kinetic energy; B and C are asymptotes for high and low μ/ζ_n ratios respectively

The change in the kinetic energy of the cantilever beam pictured in figure 1b, excited at its third natural frequency is shown in figure 6 as a function of the mass ratio divided by the neutraliser's damping ratio. The beam is excited at x/L=0.2 and the neutraliser is positioned at the free-end and has a mass ratio of 0.1; the beam and neutraliser damping ratios are set at 0.001. The actual change in the kinetic energy was calculated using equation (1) and a normalised version of equation (2), and is labelled A; the normalised minimum kinetic energy was calculated using equation (1) and equation (8) normalised by the kinetic energy of the third mode, and is labelled B; the sloping line was calculated using equation (1) and equation (9) with the 1 in the denominator neglected, and is labelled C. The optimum ratio μ/ζ_n is given by:

$$\left.\frac{\mu}{\zeta_n}\right|_{\text{opt}} = \frac{2\varphi_m(m_f)}{\varphi_m(x_n)\omega_m^2\sqrt{\mathbf{b}^{\mathrm{H}}\mathbf{b}}} \quad (10)$$

where $\mathbf{b} = m\mathbf{q}_{\min}$, and μ is the ratio of the mass of the neutraliser to the mass of the m-th mode.

It can be seen that this is quite a good approximation, and is thus considered useful in the design of a vibration neutraliser for a particular structure. It is interesting to note that as the neutraliser is moved closer to the source the maximum reduction in the kinetic energy increases. However, to achieve the maximum reduction, the optimum μ/ζ_n ratio increases, which means that either the mass of the neutraliser has to increase or the damping in the neutraliser has to decrease.

It is tempting to reduce the damping in the neutraliser, because adding mass in a vibration control device is usually not encouraged. However, it is well known that the separation between the peaks either side of the operating frequency is governed by the mass ratio [2], and clearly it is desirable to have a "reasonable" peak separation so that the system is reasonably robust to rapid changes in excitation frequency as discussed by Brennan [4]. The frequency separation between the peaks normalised to the natural frequency of the beam between the peaks is given by [17]:

$$\Delta\Omega = \varphi_m(x_n)\mu^{\frac{1}{2}} \quad (11)$$

Thus, by using equations (10) and (11) the optimum neutraliser mass and damping ratio can be determined for global vibration control once a tolerable peak separation has been defined, and this will be dependent upon the control delay in the neutraliser as discussed by Brennan [4].

3.Comparison between active control and control using a tunable vibration neutraliser

Global control of vibration can be compared using (a) a passive device with no restrictions on the passive elements (optimal passive control), (b) a tunable vibration neutraliser, and (c) fully active control, by using the theory described by Nelson and Elliott [19] for the active control of sound. The problem is cast with the attached dynamic stiffness of the control device being the control variable. With strategy (a), the imaginary part of the dynamic stiffness is constrained to be positive, with strategy (b) the imaginary part is constrained to be positive and the real part is set to infinity and with strategy (c), there are no constraints on the dynamic stiffness.

Equation (2) can be written as:

$$\mathbf{q} = \mathbf{d} + \mathbf{C}\left(\frac{k_n}{1 + k_n\alpha_{nn}}\right) \quad (12)$$

where:

$$\mathbf{d} = \mathbf{A}\varphi(x_f)f, \quad \text{and} \quad \mathbf{C} = -\mathbf{A}\varphi(x_n)\alpha_{nf}f \quad (13a,b)$$

When equation (12) is substituted into equation (1) to give the kinetic energy, the resulting equation is of hermitian quadratic form, which has a minimum when [19]:

$$k_{n(\mathrm{opt})} = \frac{-[\mathbf{C}^{\mathrm{H}}\mathbf{C}]^{-1}\mathbf{C}^{\mathrm{H}}\mathbf{d}}{1 + \alpha_{nn}[\mathbf{C}^{\mathrm{H}}\mathbf{C}]^{-1}\mathbf{C}^{\mathrm{H}}\mathbf{d}} \qquad (14)$$

and the kinetic energy is found by setting k_n to $k_{n(\mathrm{opt})}$ in equation (12) which in turn is substituted into equation (1). The optimum dynamic stiffness has real and imaginary parts which are plotted in figures 7a and b respectively for the cantilever beam used in the earlier simulations with the dynamic stiffness placed at the free-end of the beam. It can be seen that both the real and imaginary parts are both positive and negative depending upon frequency. The interpretation of these graphs is as follows. The real part of the dynamics stiffness is related to the reactive passive elements of mass and stiffness. A positive real part is means that the dynamic stiffness should be stiffness-like, and conversely a negative real part corresponds to a mass-like dynamic stiffness, and this is shown in figure 7a. If the real part of the optimum dynamic stiffness is infinite then this corresponds to a vibration neutraliser, and if it is zero then this corresponds to no mass or stiffness. The imaginary part of the dynamic stiffness corresponds to damping. If it is positive then this means that the device should absorb energy, and if it is negative then it should supply energy to the structure. It can be seen from figure 7b that at some frequencies then energy needs to be supplied to the beam, but at other frequencies the device should absorb energy. However, it should be noted that the imaginary part of the dynamic stiffness is several orders of magnitude less than the real part, and when it is set to zero for the example considered in this article, it makes little difference to the resulting kinetic energy of the beam. Thus, if an active control system is used to control global vibration, then over most frequencies it effectively has to synthesise an attached mass or a stiffness. At certain frequencies it has to synthesise a tuned vibration neutraliser or nothing at all.

From these simulations it is clear why a de-tuning control strategy has been used to control global vibration. However, this is not a simple strategy to implement in practice, as discussed above, and so it is worthwhile to see how effective a tuned vibration neutraliser is compared with optimal control,

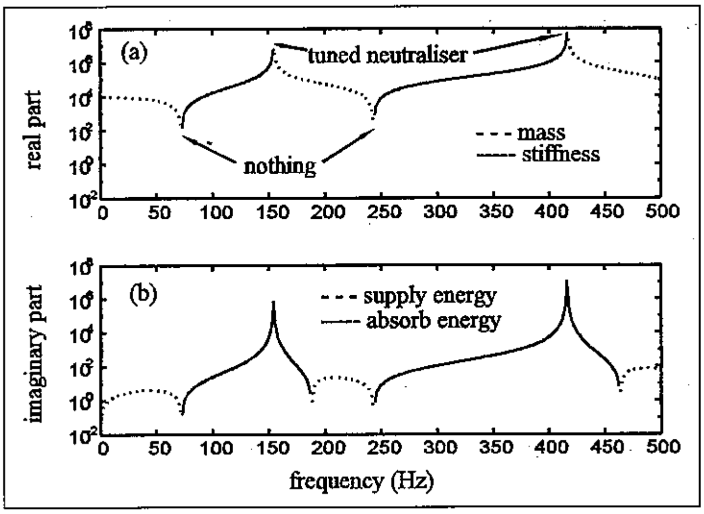

Figure 7. Optimum dyanic stiffness of the neutraliser as a function of frequency (a) is the real part and (b) is the imaginary part. Solid line – positive; dashed line – negative

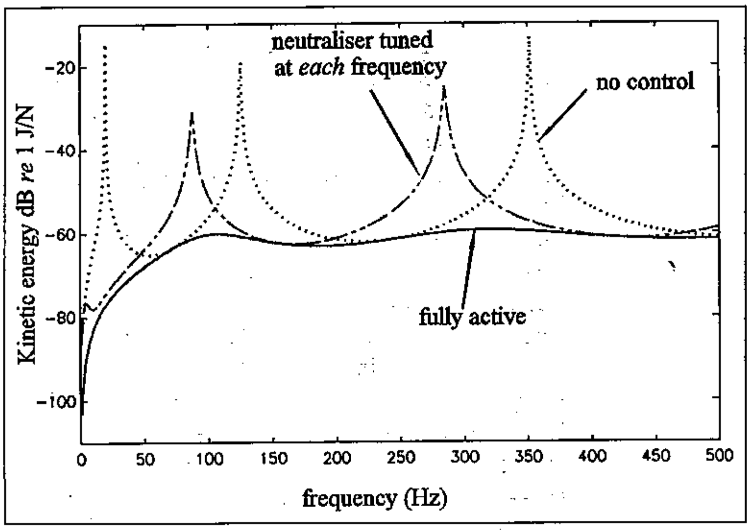

Figure 8. Comparison of the kinetic energy of the cantilever beam between fully active control and control using a tuned vibration neutraliser tuned at each frequency

but with the imaginary part constrained to be positive. Figure 8 shows the kinetic energy calculated at each frequency of the cantilever beam with the two control strategies implemented; the kinetic energy before control is also shown. It can be seen that the constrained optimal control strategy reduces the kinetic energy at most frequencies, and never makes the situation worse. The best control is achieved around the original resonance frequencies of the beam, and there are some frequencies when no control is possible. It is also clear that although the tuned vibration neutraliser is effective at some frequencies, it makes global vibration worse at other frequencies, as discussed previously. If we divide the kinetic

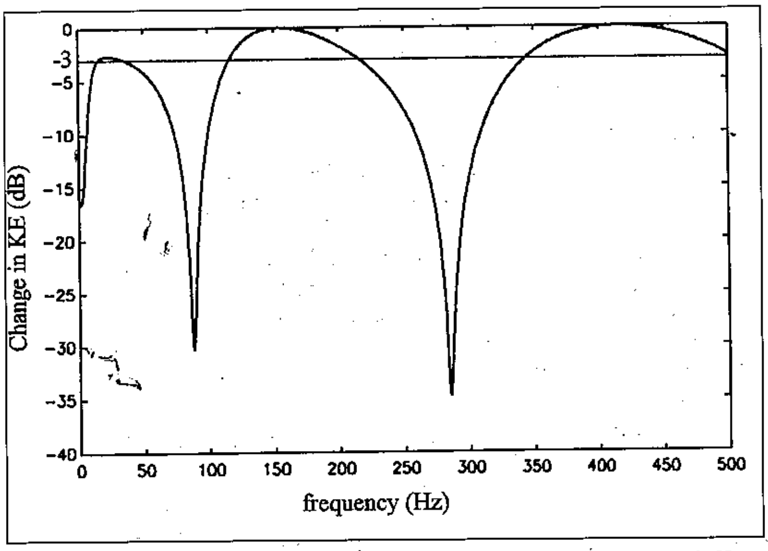

Figure 9. Ratio of the kinetic energy of the beam with fully active control to the kinetic energy of the beam controlled with a neutraliser, tuned at each frequency

energy of the beam with an optimally controlled device attached by the kinetic energy of the beam with a tuned (at each frequency) neutraliser attached, the frequency range over which the tuned neutraliser is effective can be clearly identified. This is shown in figure 9. There are large frequency ranges, for example from about 120-220 Hz and 340-500 Hz where the tunable vibration neutraliser's effectiveness is within 3 dB of the optimal passive control. These frequency ranges can be adjusted by careful choice of neutraliser position as discussed in section 2. Thus it is possible to use a resonant vibration neutraliser tuned using a local control strategy to control the global vibration of a structure. This control strategy, although sub-optimal, gives results which are very similar to a de-tuning control strategy, which requires a more sophisticated control system.

4. Conclusions

In this article the use of a tunable vibration neutraliser has been investigated for the control of global vibration of a structure. With local vibration control, where the aim is for the vibration neutraliser to pin the structure at the point of attachment, the dynamic stiffness of the neutraliser is required to be as large as possible. However, with global control the required dynamic stiffness of the neutraliser has an optimum (threshold) value, beyond which any increase does not result in improved performance. A tuned vibration neutraliser can, at some frequencies, result in an increase rather than decrease in global vibration. These frequencies are the natural frequencies of the host structure when it is pinned at the neutraliser attachment point. If these natural frequencies coincide with a frequency of interest then they can be shifted to other frequencies by changing the position of the neutraliser. By correctly positioning the neutraliser it can be tuned to be a resonant device and effect global control of the structure, that offers a performance within 3 dB of that achievable with optimal passive control.

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