Optimum tuning of a vibration neutralizer for global vibration control

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Abstract: This paper is concerned with optimization of the natural frequency of a vibration neutralizer to minimize the global vibration of a structure at a single frequency. The optimization is carried out using a quadratic minimization technique to determine the dynamic stiffness of the control device that is required, and then the optimum resonance frequency of the neutralizer is determined. It is shown that, with the exception of very low frequencies, an optimally adjusted vibration neutralizer can be as effective as an active device at a single frequency. Simulations are presented with a single neutralizer on a beam to facilitate insight into the mechanisms of control.

Keywords: vibration, absorbers, neutralizers, adaptive–passive control

NOTATION

$\mathbf{A}$	matrix of complex modal amplitudes
C	damping coefficient
f	force
$\boldsymbol{f}$	vector of forces
$\boldsymbol{g}$	vector of generalized forces
i	$\sqrt{-1}$
$\mathbf{I}$	identity matrix
J	number of neutralizers
k	stiffness
K	dynamic stiffness
$\mathbf{K}$	matrix of dynamic stiffnesses
L	length of beam
M	neutralizer mass
N	number of modes of the host structure
$\boldsymbol{q}$	vector of modal displacements of the host structure
w	lateral displacement
$\boldsymbol{w}$	vector of lateral displacements
x	position on the structure
α	frequency normalized to the natural frequency of the neutralizer
β	receptance of the host structure
ζ	neutralizer damping ratio
μ	mass ratio
$\boldsymbol{\Phi}$	vector of normalized mode shapes of the host structure
$\boldsymbol{\Psi}$	matrix of mode shapes of the host structure
ω	circular forcing frequency

Subscripts

a	neutralizer position
d	neutralizer array
f	primary force position
h	host structure
j	neutralizer index
n	mode index
o	optimum
p	primary
s	secondary

Superscripts

H	Hermitian transpose
T	transpose
$\Re(\,)$	real part
$\Im(\,)$	imaginary part

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1 INTRODUCTION

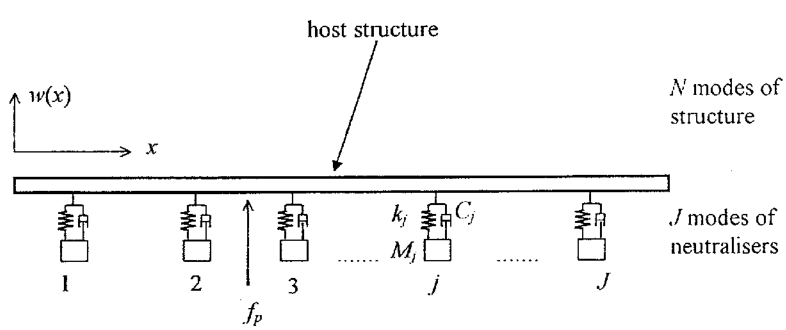
Vibration neutralizers have been used in many applications to reduce structural vibration. These range from small hand-held tools such as an electrical clipper [1, 2], to a giant static structure such as a high-rise building [3–5] or large moving structures such as aircraft [6–8]. In early applications, the natural frequency of the neutralizer was fixed to a predetermined value. However, with this condition it is possible for the device to increase rather than attenuate the vibration of the host structure if there is a change in the excitation frequency, or if the characteristics of the neutralizer change with time. Therefore, efforts have been made to prevent this by making the natural frequency of the neutralizer adjustable. Changes in the forcing frequency are tracked so that the neutralizer is tuned in such a way that the natural frequency of the neutralizer is always coincident with this frequency [9]. Recently, it has been found that a tuned neutralizer can increase the global response of a continuous structure at the forcing frequency, and that it is sometimes better to detune the neutralizer if the host structure is being forced at an off-resonance frequency [10–13].

This paper is an extension of the work on the optimization of a neutralizer for global vibration control of a continuous structure [14]. A method is presented for determining the optimum natural frequency of the neutralizer that minimizes the global vibration of a host structure. It is shown that, above a certain frequency, and provided a certain procedure to adjust the stiffness of the neutralizer is adopted, it can be as effective as fully active control. This is demonstrated on a simply supported beam.

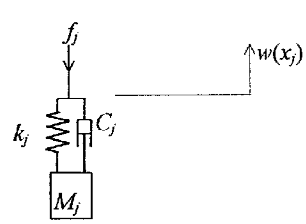
The paper is organized as follows. Following this introduction, an analytical model of a number of neutralizers on a one-dimensional structure is developed in Section 2. The dynamic behaviour of a neutralizer is studied in Section 3, and a procedure to optimize the neutralizer characteristics is presented in Section 4. Some simulations are presented in Section 5 with a single neutralizer on a beam, and then the paper ends in Section 6 with some concluding remarks.

2 MODEL FORMULATION

Consider a host structure with J tuneable vibration neutralizers attached at $x_1, x_2, \ldots, x_j, \ldots, x_J$, which is excited by a harmonic (primary) force of amplitude f_p, as shown in Fig. 1a. Although the analysis is shown for a one-dimensional system, there is no loss of generality, and the results of the analysis are applicable to a general structure. Parameters C_j, k_j and M_j are the damping coefficient, spring constant and mass of the jth neutralizer respectively. The damping coefficient of the jth neutralizer is given by $2\zeta_j\sqrt{M_jk_j}$, where ζ_j is the damping ratio of the jth neutralizer. Parameter f_j is the feedback or reaction force from the jth neutralizer when the whole system vibrates, and is shown in Fig. 1b. The global vibration of the host structure can be characterized by its kinetic energy, and its response is calculated at each single frequency of the excitation force. To evaluate the performance of the vibration neutralizers, the



(a) J number of neutralisers attached to an arbitrary structure



(b) The j-th neutraliser

Fig. 1 Schematic diagram representing J neutralizers attached to an arbitrary structure that has N modes in the frequency range of interest kinetic energy of the host structure when using optimum *feedforward* active control is used as a benchmark. This active control technique, which illustrates the performance limits of any active control system for a given set of actuators, is also assumed to be the performance limit of control using tuneable vibration neutralizers. The response of the structure is approximated by the superposition of N modes for both active control and control using tuneable vibration neutralizers. The issue that is addressed in this paper is whether a passive control system such as a vibration neutralizer can perform as well as an active system at a single frequency if it is correctly optimized.

The displacement of the host structure at any point x is given by

$$w(x) = \boldsymbol{\Phi}^{\mathrm{T}}(x)\boldsymbol{q} \tag{1}$$

where $w(x)$ is the displacement at location x, $\boldsymbol{\Phi}(x)$ is the $N \times 1$ vector of the normalized mode shapes and $\boldsymbol{q}$ is the $N \times 1$ vector of modal displacement amplitudes of the structure respectively. The superscript T denotes the transpose of the vector. The modal displacement vector is given by

$$\boldsymbol{q} = \mathbf{A}\boldsymbol{g} \tag{2}$$

where $\mathbf{A}$ is the $N \times N$ diagonal matrix of complex modal amplitudes and $\boldsymbol{g}$ is the $N \times 1$ vector of generalized forces acting on the structure. The nnth element of the complex modal amplitude matrix $\mathbf{A}$ is given by

$$A_n = \frac{1}{M_{\mathrm{h}n}(\omega_{\mathrm{h}n}^2 - \omega^2 + \mathrm{i}2\zeta_{\mathrm{h}n}\omega\omega_{\mathrm{h}n})} \tag{3}$$

where $M_{\mathrm{h}n}$, $\omega_{\mathrm{h}n}$ and $\zeta_{\mathrm{h}n}$ are the modal mass, the nth circular natural frequency and the nth modal damping ratio of the host structure, and ω is the circular excitation frequency. The vector of generalized forces can be written as

$$\boldsymbol{g} = \boldsymbol{g}_{\mathrm{p}} + \boldsymbol{g}_{\mathrm{d}} \tag{4}$$

where $\boldsymbol{g}_{\mathrm{p}}$ and $\boldsymbol{g}_{\mathrm{d}}$ are the vector of generalized primary forces and the vector of generalized forces from the neutralizer array respectively. The vector of forces from the neutralizer array has the form

$$\boldsymbol{g}_{\mathrm{d}} = \boldsymbol{\Psi}\boldsymbol{f}_{\mathrm{d}} \tag{5}$$

where $\boldsymbol{\Psi}$ is the $N \times J$ matrix of the structural normalized mode shape evaluated at the location of each neutralizer, and $\boldsymbol{f}_{\mathrm{d}}$ is the $J \times 1$ vector of feedback forces from the neutralizers. $\boldsymbol{\Psi}$ and $\boldsymbol{f}_{\mathrm{d}}$ are respectively given by

$$\boldsymbol{\Psi} = [\boldsymbol{\Phi}(x_1)\boldsymbol{\Phi}(x_2)\cdots\boldsymbol{\Phi}(x_j)\cdots\boldsymbol{\Phi}(x_J)] \tag{6}$$

$$\boldsymbol{f}_{\mathrm{d}} = -\mathbf{K}\boldsymbol{w}(x_j) \tag{7}$$

where $\mathbf{K}$ is the $J \times J$ diagonal matrix of neutralizer dynamic stiffness and $\boldsymbol{w}(x_j)$ is the $J \times 1$ displacement vector of the host structure evaluated at the location of the jth neutralizer. The dynamic stiffness of the jth neutralizer, K_j, is the jjth element of matrix $\mathbf{K}$ and is given by **[15]**

$$K_j = -M_j\omega^2\left(\frac{1 + \mathrm{i}2\zeta_j\alpha_j}{1 - \alpha_j^2 + \mathrm{i}2\zeta_j\alpha_j}\right) \tag{8}$$

where $\alpha_j = \omega/\omega_j$ is the tuning ratio of the jth neutralizer, ω_j being the natural frequency of the jth neutralizer given by $(k_j/M_j)^{1/2}$. Using equation (1), equation (7) can be written as

$$\boldsymbol{f}_{\mathrm{d}} = -\mathbf{K}\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{q} \tag{9}$$

By combining equations (4), (5) and (9), the vector of modal displacement amplitudes of the host structure can be written as

$$\boldsymbol{q} = [\mathbf{I} + \mathbf{A}\boldsymbol{\Psi}\mathbf{K}\boldsymbol{\Psi}^{\mathrm{T}}]^{-1}\mathbf{A}\boldsymbol{g}_{\mathrm{p}} \tag{10}$$

This can be used to find the kinetic energy of the structure, which is given by **[16]**

$$\mathrm{KE} = \frac{M_{\mathrm{h}}\omega^2}{4}\boldsymbol{q}^{\mathrm{H}}\boldsymbol{q} \tag{11}$$

where the superscript H denotes the Hermitian transpose and M_{h} is the mass of the host structure. The kinetic energy of the host structure without neutralizers attached can be determined by simply setting all elements of $\mathbf{K}$ in equation (10) to zero.

3 DYNAMIC BEHAVIOUR OF THE NEUTRALIZER

The dynamic stiffness of the jth neutralizer given in equation (8) can be written in terms of its real and imaginary parts, given by

$$\Re(K_j) = -M_j\omega^2\left[\frac{1 - \alpha_j^2 + 4\zeta_j^2\alpha_j^2}{(1 - \alpha_j^2)^2 + 4\zeta_j^2\alpha_j^2}\right] \tag{12}$$

and

$$\Im(K_j) = \frac{2M_j\zeta_j\omega^2\alpha_j^3}{(1 - \alpha_j^2)^2 + 4\zeta_j^2\alpha_j^2} \tag{13}$$

where $\Re(K_j)$ and $\Im(K_j)$ are the real and imaginary parts of the dynamic stiffness respectively. Now, the real part of the dynamic stiffness of the neutralizer can be positive or negative depending on the value of the tuning ratio, α_j, but the imaginary part is constrained to be positive for all values of α_j because a neutralizer can only absorb energy. The dynamic stiffness of the neutralizer changes its form with frequency and can be approximated by the following expressions:

When $\alpha_j \ll 1$:

$$K_j \approx -M_j\omega^2 \tag{14a}$$

When $\alpha_j = 1$:

$$K_j \approx \frac{\mathrm{i}M_j\omega^2}{2\zeta_j} \tag{14b}$$

When $\alpha_j \gg 1$:

$$K_j \approx k_j \tag{14c}$$

Thus, when the natural frequency of the neutralizer is much greater than the forcing frequency ($\alpha_j \ll 1$), its dynamic stiffness is dominated by its mass. When the natural frequency of the neutralizer coincides with the forcing frequency ($\alpha_j = 1$), its dynamic stiffness is largely influenced by its damping, and when the natural frequency of the neutralizer is much lower than the forcing frequency ($\alpha_j \gg 1$), its dynamic stiffness is dominated by its spring constant. For these reasons, when $\Re(K_j)$ is negative ($\alpha_j < 1$), the neutralizer is said to behave like a mass, when $\alpha_j = 1$ the neutralizer is said to behave like a damper and when $\Re(K_j)$ is positive ($\alpha_j > 1$) the neutralizer is said to behave like a stiffness.

4 OPTIMUM NEUTRALIZER TUNING RATIO

Examination of the dynamic stiffness of a neutralizer given in equation (8) shows that it depends upon three parameters, namely its mass, M_j, its damping ratio, ζ_j, and its tuning ratio, α_j. Thus, for an optimum neutralizer that minimizes the response of the host structure, all of these parameters have to be optimized. For a neutralizer that is tuned so that the natural frequency is coincident with the forcing frequency, it has been found that there is a threshold value of the mass ratio (mass ratio = mass of neutralizer/modal mass of host structure) divided by the neutralizer damping ratio, beyond which no further reduction in the global vibration of the host structure can be achieved [**14**]. It is shown in Section 5 that the optimum mass of the optimally detuned neutralizer can also be determined using a similar method, so only the optimum tuning ratio of the neutralizer is determined in this section. Because feedforward active control is assumed to be the performance limit of the control using a tuneable neutralizer, the optimum tuning ratio of the neutralizer is determined partly by using this technique.

Consider a similar structure to that in Fig. 1 but, instead of neutralizers being fitted, J secondary forces are applied in the same positions, as shown in Fig. 2. The vector of modal amplitudes of the structure can be written in terms of its complex amplitude matrix $\mathbf{A}$ and the generalized forces as

$$\boldsymbol{q} = \mathbf{A}(\boldsymbol{g}_\mathrm{p} + \boldsymbol{\Psi}\boldsymbol{f}_\mathrm{s}) \tag{15}$$

where $\boldsymbol{f}_s$ is the J-length vector of the amplitudes of the secondary forces. Following the work by Nelson and Elliott [**16**], equation (15) can be expressed as

$$\boldsymbol{q} = \boldsymbol{d} + \mathbf{G}\boldsymbol{f}_\mathrm{s} \tag{16}$$

where

$$\boldsymbol{d} = \mathbf{A}\boldsymbol{g}_\mathrm{p} \tag{17a}$$

$$\mathbf{G} = \mathbf{A}\boldsymbol{\Psi} \tag{17b}$$

Substituting for $\boldsymbol{q}$ in equation (11) and expanding gives the kinetic energy in standard Hermitian quadratic

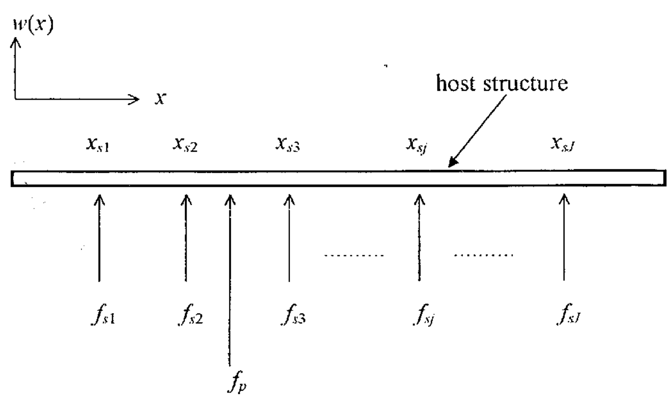


Fig. 2 One-dimensional structure with fully active control: f_p is the primary force, $f_{\mathrm{s}1}, f_{\mathrm{s}2}, \ldots, f_{\mathrm{s}J}$ are the secondary forces, and $x_{\mathrm{s}1}, x_{\mathrm{s}2}, \ldots, x_{\mathrm{s}J}$ are the location of the forces on the host structure

form as

$$KE = \frac{M_h\omega^2}{4} \times \{f_s^H G^H G f_s + f_s^H G^H d + d^H G f_s + d^H d\} \quad (18)$$

The kinetic energy has a minimum value when the vector of the secondary forces is [16]

$$f_{so} = -[G^H G]^{-1} G^H d \quad (19)$$

The corresponding optimum vector of modal amplitudes is given by

$$q_o = [I - G[G^H G]^{-1} G^H] d \quad (20)$$

Equating the optimum vector of secondary forces to the feedback forces from the neutralizers given in equation (9) results in

$$-K_o \Psi^T q_o = f_s \quad (21)$$

where K_o is a diagonal matrix whose jjth entry is the optimum dynamic stiffness of each neutralizer, which can be written as

$$K_{oj} = -f_{soj}(\Phi^T(x_j) q_o)^{-1} \quad (22)$$

Thus, there is an equivalent dynamic stiffness of a control device that minimizes the kinetic energy of the host structure. Now, the real and imaginary parts of the optimum secondary forces calculated using equation (19) can have both positive and negative values. Although the real part of each neutralizer dynamic stiffness has no constraints (it can be positive or negative), the imaginary part is constrained to be positive only because a neutralizer can only absorb energy. Thus, at a single frequency the optimum neutralizer can be determined by setting the real part of the neutralizer dynamic stiffness to the real part of equation (22) and setting the imaginary part of the neutralizer dynamic stiffness to the positive imaginary part of equation (22). This could be done by changing both stiffness and damping as discussed by Kidner and Brennan [17, 18]. If only the stiffness, i.e. the tuning ratio, can be adjusted, then it is sensible to optimize the real part of the neutralizer dynamic stiffness only.

Rearranging equation (12) and solving for the square of the tuning ratio, α_j^2, gives

$$\alpha_j^2 = \frac{4R_2\zeta_j^2 - 4R_1\zeta_j^2 + 2R_1 - R_2 \pm \sqrt{(4R_1\zeta_j^2 - 4R_2\zeta_j^2 - 2R_1 + R_2)^2 - 4R_1(R_1 - R_2)}}{2R_1} \quad (23)$$

where $R_1 = \Re\{K_{jo}\}$ and $R_2 = M_j\omega^2$. As the damping in the neutralizer is normally small, it can be neglected and equation (23) reduces to

$$\alpha_{jo} = \sqrt{1 - \frac{M_j\omega^2}{\Re\{K_{jo}\}}} \quad (24)$$

which is the optimum tuning ratio of the jth neutralizer. If the tuning ratio is unity, then the neutralizer is tuned so that its natural frequency is coincident with the forcing frequency. Substituting the optimum tuning ratio from equation (24) into equation (8) gives the optimally adjusted dynamic stiffness of the neutralizer as

$$K_{jo} = -M_j\omega^2\left(\frac{1 + i2\zeta_j\alpha_{jo}}{1 - \alpha_{jo}^2 + i2\zeta_j\alpha_{jo}}\right) \quad (25)$$

By examining equations (24) and (25), some physical insight can be gained into the optimum neutralizer characteristics:

1. When $\Re\{K_{jo}\} < 0$, the tuning ratio is greater than 1. Therefore, the neutralizer is detuned and the natural frequency of the neutralizer is lower than the forcing frequency, so the neutralizer behaves like a stiffness.
2. When $\Re\{K_{jo}\} > M_j\omega^2$, the tuning ratio is less than 1. Therefore, the neutralizer is detuned but the natural frequency of the neutralizer is greater than the forcing frequency, so the neutralizer behaves like a mass.
3. When $\Re\{K_{jo}\} = M_j\omega^2$, the tuning ratio is zero which means no neutralizer is required.
4. When $\Re\{K_{jo}\} = \pm\infty$, the tuning ratio is unity. Therefore, the neutralizer is tuned and the natural frequency of the neutralizer coincides with the forcing frequency, so the neutralizer behaves like a damper.
5. When $0 \leqslant \Re\{K_{jr}\} < M_j\omega^2$, the tuning ratio is not physical because it is imaginary.

Condition (5) occurs, if the mass is too large at the frequency of interest.

If multiple neutralizers are applied to a structure, then at any single frequency the optimum tuning of all the neutralizers may be different. Some neutralizers may be tuned so that they are resonant, and some may be tuned so that they appear mass or stiffness like, as discussed by Huang and Fuller [13] and Dayou [19].

To tune a neutralizer so that its tuning ratio is optimum is more difficult in practice than simply tuning a neutralizer so that it is resonant at the forcing frequency. A simple procedure using only two accelerometers can be used to tune the device so that it is resonant [20]. Moreover, if multiple tuned neutralizers are used, then a decentralized control strategy can be adopted. However, to estimate the kinetic energy of a host structure, several accelerometers are required, which can significantly complicate the control system, especially if multiple neutralizers are used, as all sensors will have to be connected to all neutralizers. This is the disadvantage of using 'detuned' neutralizers.

5 SIMULATIONS

To demonstrate the formulation developed above, a series of computer simulations is presented on the use of a single neutralizer to reduce the kinetic energy of a simply supported beam. The beam is excited by a single harmonic point force f_p at x_f, and the neutralizer is fitted at position x_a. When a single neutralizer is attached, the vector of modal amplitudes given by equation (10) can be written in an alternative form that avoids the matrix inverse operation [**14**]:

$$\boldsymbol{q} = \mathbf{A} f_p \left[\boldsymbol{\Phi}(x_f) - \boldsymbol{\Phi}(x_a) \left(\frac{K_a \beta_{af}}{1 + K_a \beta_{aa}} \right) \right] \tag{26}$$

where $\boldsymbol{\Phi}(x_f)$, $\boldsymbol{\Phi}(x_a)$, β_{aa} and β_{af} are the vector of normalized mode shapes of the beam evaluated at the point force location, the vector of normalized mode shapes of the beam evaluated at the neutralizer location, the point receptance of the host structure at the location of the neutralizer and the transfer receptance of the host structure between the neutralizer position and the primary force respectively. Again, following the work by Nelson and Elliott [**16**], equation (26) can be rewritten as

$$\boldsymbol{q} = \mathbf{B}_1 + \mathbf{B}_2 \left(\frac{K_a}{1 + K_a \beta_{aa}} \right) \tag{27}$$

where

$$\mathbf{B}_1 = \mathbf{A} \boldsymbol{\Phi}(x_f) f_p \tag{28a}$$

$$\mathbf{B}_2 = -\mathbf{A} \boldsymbol{\Phi}(x_a) \beta_{af} f_p \tag{28b}$$

Therefore, the standard Hermitian formulation for the beam kinetic energy can be formed which has a minimum when [**14**]

$$K_o = \frac{-[\mathbf{B}_2^H \mathbf{B}_2]^{-1} \mathbf{B}_2^H \mathbf{B}_1}{1 + \beta_{aa} [\mathbf{B}_2^H \mathbf{B}_2]^{-1} \mathbf{B}_2^H \mathbf{B}_1} \tag{29}$$

K_o is the optimum dynamic stiffness that has to be generated by a single control device. Substitution of the real part of equation (29) into equation (24) gives the optimum tuning ratio of the neutralizer, and combining equations (11), (25) and (26) gives the kinetic energy of the beam with an optimally adjusted neutralizer. The kinetic energy of the beam with active control can be calculated by combining equations (11), (19) and (20).

The simply supported beam of length L used in the simulations has dimensions 1 m × 0.0381 m × 0.00635 m, and the material density and Young's modulus are 7870 kg/m^3 and 207 GN/m^2 respectively. The frequency of interest is set from 15 to 450 Hz, and the neutralizer is fitted at $x_a = 9L/20$, which does not coincide with any nodal points in the frequency of interest.

To determine the optimum mass of the neutralizer, a numerical method is suggested. If the mass of the optimally detuned neutralizer is allowed to increase for a given damping ratio, the kinetic energy of the host structure at each frequency will decrease to a certain value and then become roughly constant. This can be seen in Fig. 3 for selected frequencies. Here the kinetic energy of the beam is plotted against the ratio μ_a/ζ_a, where μ_a is the ratio of the neutralizer mass to the beam mass and ζ_a is the damping ratio of the single neutralizer. The optimum mass of the neutralizer is taken to be when there are no

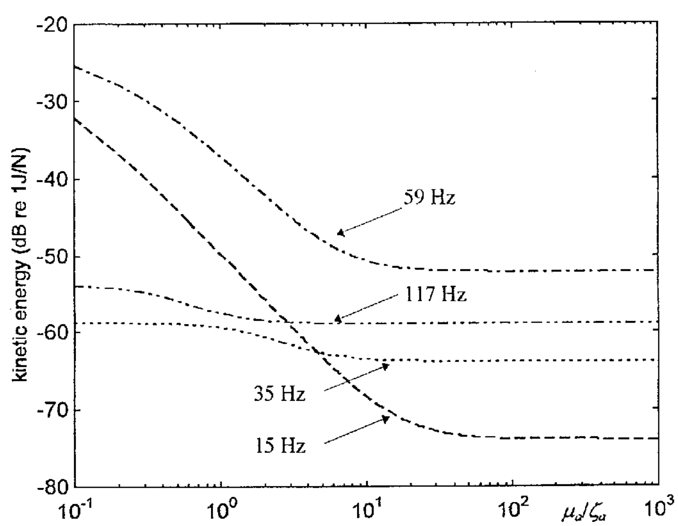


Fig. 3 Kinetic energy of a beam as a function of the ratio of mass ratio to neutralizer damping ratio at 15 and 59 Hz (first and second natural frequencies of the beam), and at 35 and 117 Hz (off-resonance frequencies). Neutralizer at $x_a = 9L/20$, $\zeta_a = 0.001$ and $x_f = 0.1L$

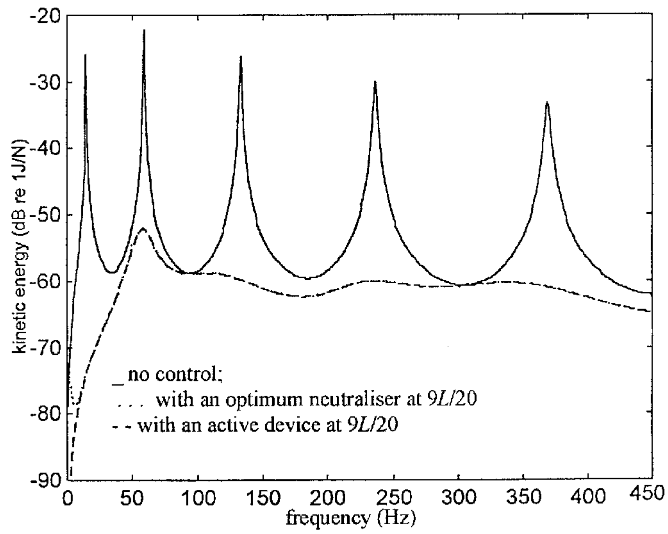


Fig. 4 Comparison between beam kinetic energy when active control is used and when an optimally adjusted neutralizer is used, both located at $x = 9L/20$: μ_a/ζ_a for the neutralizer is 40 with $\zeta_a = 0.001$, and the beam is excited by a point force at $x_a = 0.1L$

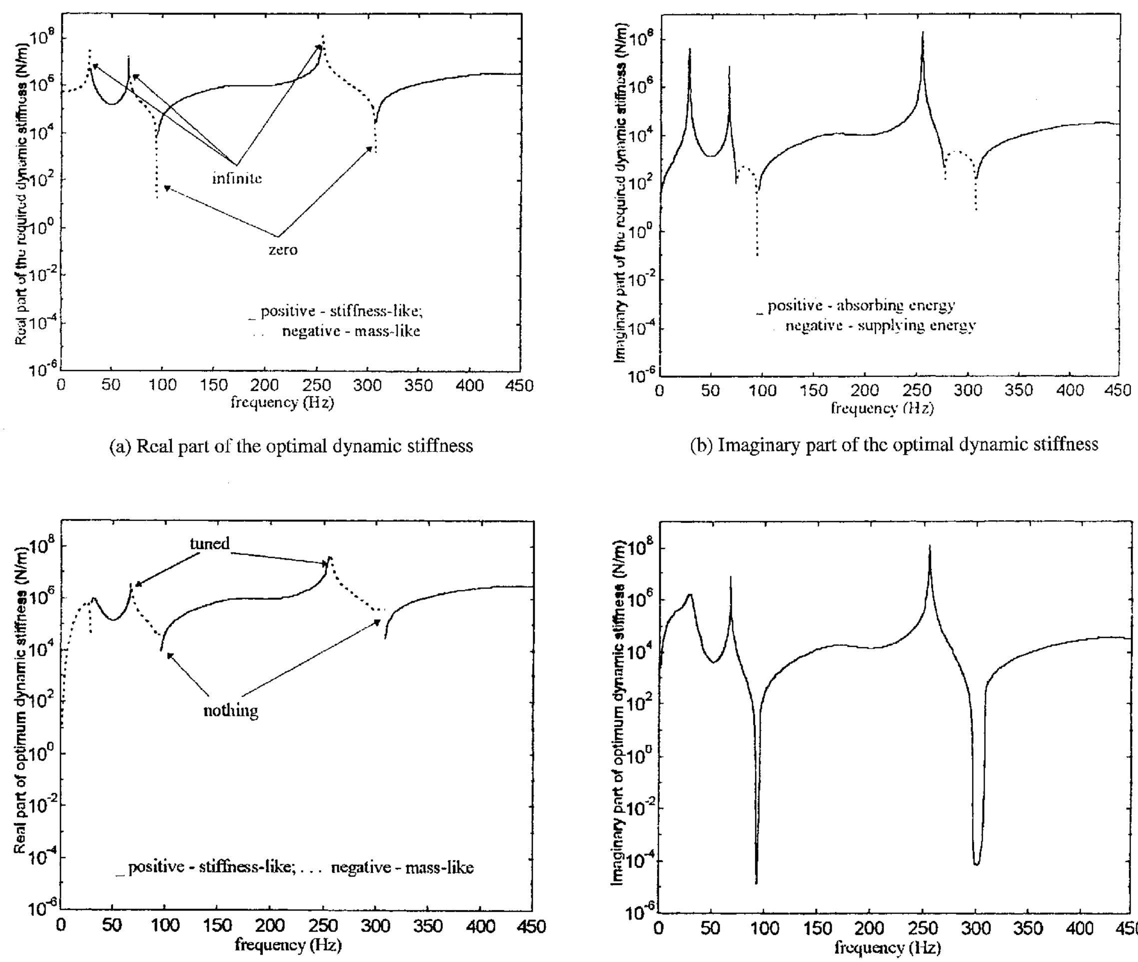
further changes in the kinetic energy of the beam. From Fig. 3 it can be seen that the optimum neutralizer mass is higher at lower frequencies. Therefore, the neutralizer optimum mass at 15 Hz can be taken as the optimum mass for the whole frequency range of interest in this case, and thus μ_a/ζ_a is about 40. It should be noted that this method of selecting the neutralizer mass may not necessarily be true under certain circumstances, and this is discussed later in this section.

Using these parameters, the kinetic energy of the beam is plotted in Fig. 4 for no control, an optimal neutralizer (at each frequency) and active control. It can be seen that the effectiveness of the neutralizer is similar to that of active control except below 15 Hz. This is because the mass of the neutralizer is optimized only at this frequency and above. In order to increase the performance of the neutralizer at lower frequencies, the mass of the neutralizer must be increased.

The mechanism of control using the tuneable vibration neutralizer can be understood by comparing the real and imaginary parts of its dynamic stiffness with the real and imaginary parts of the optimum dynamic stiffness, which are plotted in Figs 5a and b. Its real part is negative in some frequency ranges and is positive in other frequency ranges. A negative real part means that the optimum dynamic stiffness should be mass like, and a positive real part corresponds to a stiffness-like characteristic. At some frequencies, the real part has an infinite value, which corresponds to a tuned neutralizer that has damping-like characteristics. At other frequencies the optimum dynamic stiffness is zero, which means no mass or stiffness is required. The imaginary part of the optimum dynamic stiffness also has both negative and positive values. If it is positive then it means that the device should absorb energy from the beam, and if it is negative then it should supply energy to the beam.

The real and imaginary parts of the dynamic stiffness of the optimal neutralizer are shown in Figs 5c and d. It can be seen that the real part has positive and negative values. There are also frequencies where the neutralizer is tuned so that its natural frequency is coincident with the forcing frequency, and where no neutralizer is required. The imaginary part of the dynamic stiffness of the optimal neutralizer has only positive values as required. It should be noted, however, that, although the active control device is required to supply energy to the system at some frequencies, there is a negligible difference in the resultant kinetic energy of the beam when an optimal neutralizer is used.

The optimum neutralizer tuning ratio is shown in Fig. 6. Comparison with Fig. 5c shows that the tuning ratio is greater than one when the real part of the neutralizer dynamic stiffness is positive but less than one when the real part of the neutralizer dynamic stiffness is negative. At the frequency when no neutralizer is required, the tuning ratio jumps from a value close to zero to a value that is much greater than one. There is a particular frequency range of interest, which is from 300 to 308 Hz, marked A in Fig. 6. In this range, $\Re\{K_{jr}\}$ has a value between 0 and $M_j\omega^2$, which results in a purely imaginary tuning ratio. This shows that the neutralizer mass selected using the procedure suggested earlier is not



(a) Real part of the optimal dynamic stiffness

(b) Imaginary part of the optimal dynamic stiffness

(c) Real part of the optimal neutraliser dynamic stiffness

(d) Imaginary part of the optimal neutraliser dynamic stiffness

Fig. 5 Real and imaginary parts of the optimal dynamic stiffness of an active control device and the optimal neutralizer dynamic stiffness

appropriate for this particular frequency range. The best thing to do would be to remove the neutralizer or to use additional control devices, as one is insufficient.

To give further insight into the control mechanisms the displacement of the beam at several frequencies is plotted in Fig. 7. The location of the primary excitation force is labelled point A, and the position of the neutralizer or active device is labelled point B. It can be seen that at all four frequencies the optimum solution is not to pin the beam at the position of the control device (which is what a neutralizer would do if it were tuned so that its natural frequency was coincident with the forcing frequency). There is a certain motion of the beam at the attachment point of the control device, so that the overall vibration level (quantified by kinetic energy) is minimized. It can also be seen that there is also no visible difference between using an optimally adjusted neutralizer and active control.

6 CONCLUSIONS

In this paper, an optimization procedure for tuning a vibration neutralizer for global vibration control of a structure has been described. It has been shown that, in general, the optimal neutralizer is not resonant at the forcing frequency but is tuned so that the natural frequency is either lower or higher than the forcing frequency. The differences in control performance between using feed-forward active control and using an optimally adjusted

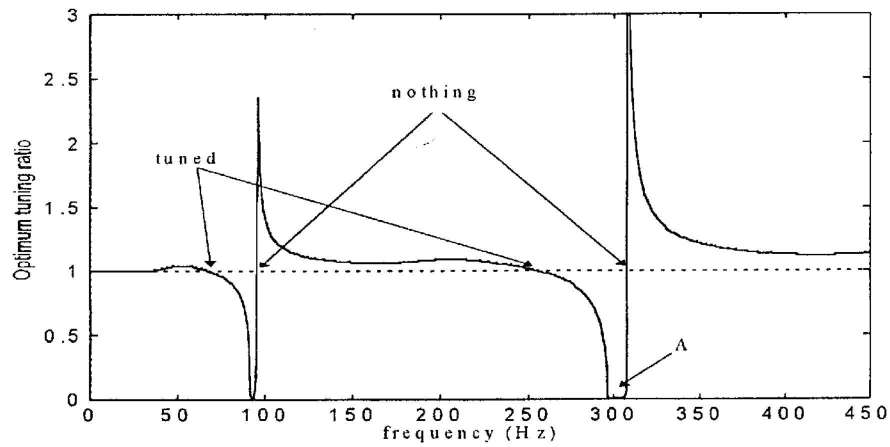


Fig. 6 Tuning ratio of the optimally adjusted neutralizer

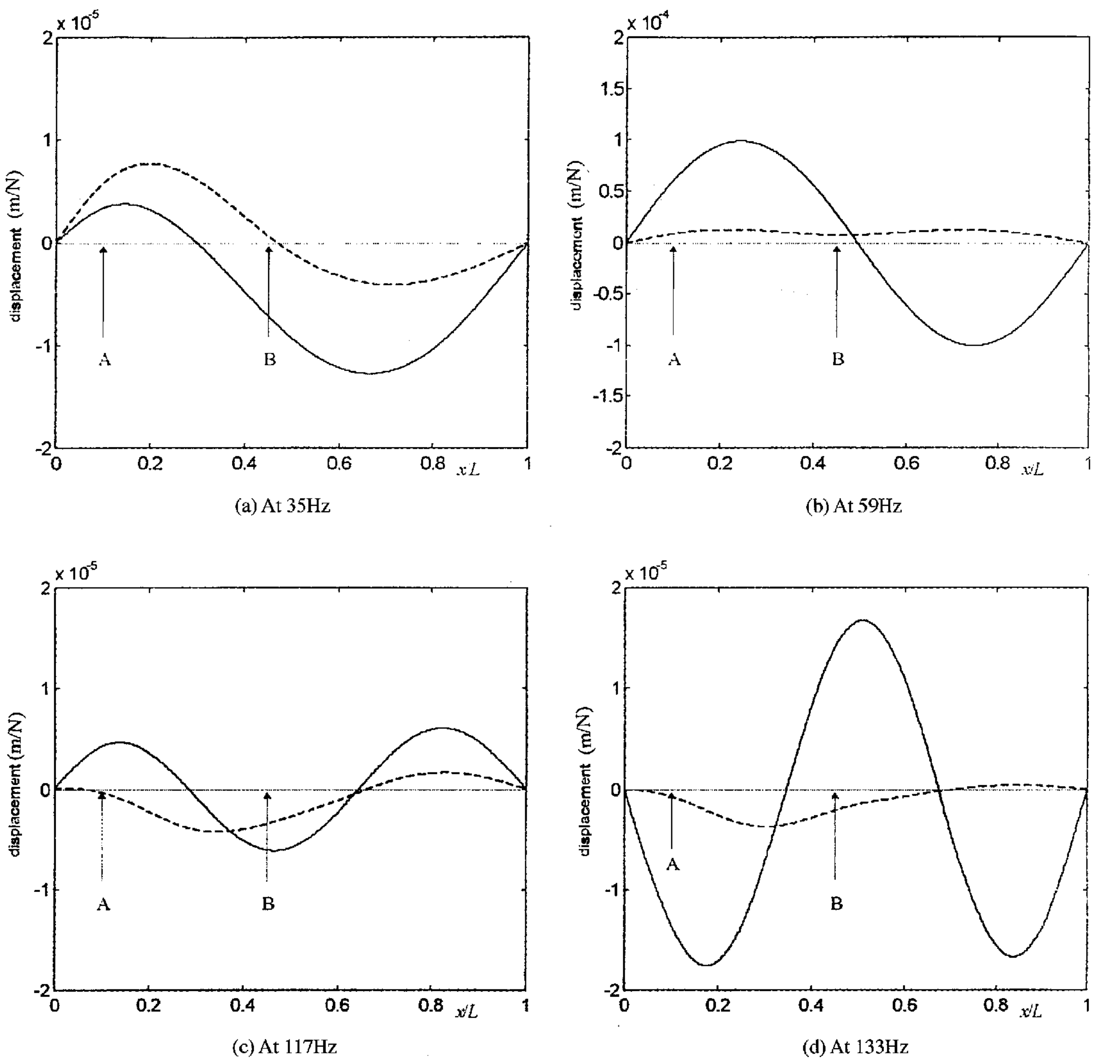


Fig. 7 Comparison of the beam displacement before and after control with an optimally adjusted neutralizer and with active control applied at $x = 9L/20$: solid line, no control; dashed line, with the optimal neutralizer or with an active device; A, location of the point primary force ($x = L/10$); B, location of the control device ($x = 9L/20$); for the neutralizer, $\mu_a/\zeta_a = 40$, $\zeta_a = 0.001$

neutralizer have been shown to be small except at very low frequencies, where an unacceptable large mass is required. Simulations have been presented using a beam as a host structure to help understanding of the physical mechanisms of global vibration control.

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