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Experimental verification of the optimal tuning of a tunable vibration neutralizer for global vibration control

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Abstract

A theoretical method has been previously proposed by the authors to optimize a tunable vibration neutralizer for global vibration control. However, experimental verification of the tuning method has yet to be presented. This paper aims to do this. It is shown that by using the proposed optimization method, the tunable vibration neutralizer can be as effective as an active control device in reducing global vibration of a structure. One particularly interesting finding is that although the vibration neutralizer is a passive device which is incapable of supplying energy to a system, it appears to be as effective as active control in reducing the global vibration of a structure, even in the frequency range where the control device is required to supply energy.

Keywords: Vibration; Absorbers; Neutralizers; Adaptive–passive control

1. Introduction

In the last decade, the tunable vibration neutralizer has been the subject of extensive research for the purpose of global vibration control. The aim has been to find an alternative method to active control, for sound and vibration problems, and for pure

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academic interest. As a result, there has been quite a lot of theoretical work published on this subject [1–9].

The most recent development on the theoretical work has been the determination of the optimal tuning of a vibration neutralizer that minimizes the kinetic energy of a host structure, which can be found in references [10–12]. The method on how to determine the optimal tuning of the neutralizer has been called the active-passive analogy [13]. In another development, a procedure on how to apply multiple tuned tunable vibration neutralizers has been suggested by the authors in reference [14]. In parallel with this, a method on how to tune a vibration neutralizer has been proposed [7]. Unfortunately, there have been no experimental results presented regarding the effectiveness of an optimized vibration neutralizer.

This paper is an extension of previous work [10], aimed at providing experimental evidence supporting the suggested optimal tuning method for a single neutralizer. It is found that the global reduction by the optimized vibration neutralizer is comparable to that of active control. One interesting result is that although the vibration neutralizer is incapable of supplying energy to a system, its effectiveness in reducing the global vibration of a host structure in the frequency range where the control device is required to supply energy is very close to the reduction achieved by a fully active control device.

This paper is arranged in 5 sections. Following this introduction there is a brief review on the proposed optimal tuning method of a vibration neutralizer for global vibration control in Section 2. The experimental method to verify the proposed tuning method is described in Section 3, and the experimental results are discussed in Section 4, which highlights several important findings. The paper is closed with some conclusions in Section 5.

2. Review on the optimal tuning method of tunable vibration neutralizer for global vibration control

2.1. Dynamic behavior of a structure with control devices attached

The dynamic behavior of a structure with a control device attached has been described in many articles for example [4–6] and [10–15]. However, the theoretical formulation is briefly described here for convenience.

Consider a general structure which is excited by harmonic primary forces of arbitrary amplitude f_p with J control devices fitted at $x_1, x_2, \ldots, x_j, \ldots, x_J$ as shown in Fig. 1. The displacement of the host structure at any point can be written in terms of a finite number of M modes as

$$w(x) = \boldsymbol{\Phi}^T(x)\mathbf{q} \tag{1}$$

where $w(x)$ is the displacement of the structure at location x, $\boldsymbol{\Phi}(x)$ is the M-length vector of the normalized mode shapes and $\mathbf{q}$ is the M-length vector of the modal

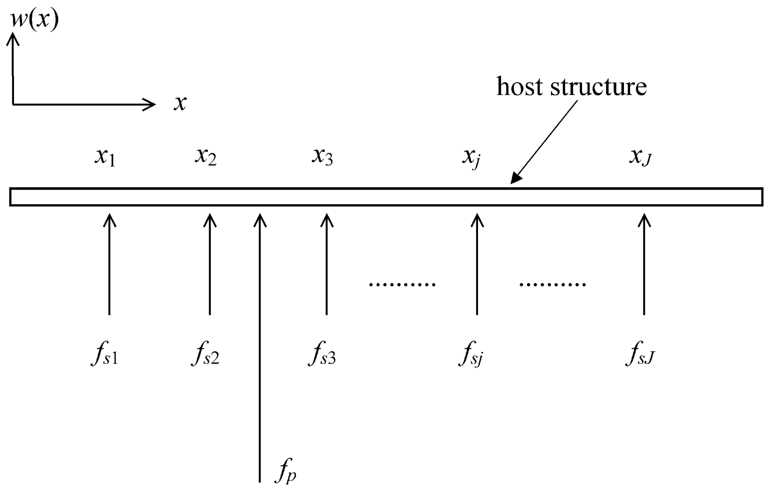


Fig. 1. An arbitrary structure with control devices attached. f_p is the primary force, $f_{s1}, f_{s2}, \ldots, f_{sJ}$ are the secondary forces, and $x_{s1}, x_{s2}, \ldots, x_{sJ}$ are the location of the forces on the host structure.

displacement amplitudes. Superscript^T denotes the transpose of the vector and the $e^{j\omega t}$ time dependence is suppressed for clarity.

If active control devices are used, the modal displacement amplitudes of the structure can be written in term of its complex amplitude matrix **A** and the generalized forces as

$$\mathbf{q} = \mathbf{A}\left(\mathbf{g}_{\mathrm{p}} + \boldsymbol{\psi}\mathbf{f}_{\mathrm{S}}\right) \tag{2}$$

where $\mathbf{g}_{\mathrm{p}}$, $\boldsymbol{\psi}$ and $\mathbf{f}_{\mathrm{s}}$ are the J-length vector of primary forces, $M \times J$ matrix of the normalized mode shapes and the J-length vector of the secondary forces amplitudes respectively. Following the work by Nelson and Elliott [15], Eq. (2) can be expressed as

$$\mathbf{q} = \mathbf{d} + \mathbf{G}\mathbf{f}_s \tag{3}$$

where

$$\mathbf{d} = \mathbf{A}\mathbf{g}_{\mathrm{p}}; \quad \mathbf{G} = \mathbf{A}\boldsymbol{\psi} \tag{4a, b}$$

The kinetic energy of the host structure can be written as

$$KE = \frac{M_{\mathrm{h}}\omega^2}{4}\mathbf{q}^H\mathbf{q} \tag{5}$$

M_{h} and ω are the mass of the host structure and the circular frequency of the primary forces.

Eq. (3) can be substituted into (5), and expanding the resulting expression gives the kinetic energy of the host structure in standard Hermitian quadratic form as

$$KE = \frac{M_h\omega^2}{4}\left\{\mathbf{f}_s^H\mathbf{G}^H\mathbf{G}\mathbf{f}_s + \mathbf{f}_s^H\mathbf{G}^H\mathbf{d} + \mathbf{d}^H\mathbf{G}\mathbf{f}_s + \mathbf{d}^H\mathbf{d}\right\} \tag{6}$$

The kinetic energy is a minimum when the vector of the secondary forces is [15]

$$\mathbf{f}_{s(\mathrm{opt})} = -\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H\mathbf{d} \tag{7}$$

and the resulting optimum vector of modal amplitudes can be written as [10]

$$\mathbf{q}_{(\mathrm{opt})} = \left[\mathbf{I} - \mathbf{G}\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H\right]\mathbf{d} \tag{8}$$

2.2. Optimal tuning of a tunable vibration neutralizer

As mentioned earlier, the optimal method for tuning a vibration neutralizer for global vibration control can be found in references [10–13], and this method is briefly described here for convenience. If all of the secondary forces in Fig. 1 are replaced with vibration neutralizers, then, the equivalent feedback forces $\mathbf{f}_d$ from the neutralizers can be written as

$$\mathbf{f}_d = -\mathbf{K}\boldsymbol{\psi}^T\mathbf{q} \tag{9}$$

where $\mathbf{K}$ is a diagonal matrix of dynamic stiffness of the neutralizers. Therefore, the secondary force vector $\mathbf{f}_s$ in Eq. (2) onwards can be replaced with the equivalent vector of neutralizer forces $\mathbf{f}_d$ given in Eq. (9). The dynamic stiffness of the individual j-th neutralizer is

$$K_j = -M_j\omega^2\left[\frac{1 + i2\zeta_j\alpha_j}{1 - \alpha_j^2 + i2\zeta_j\alpha_j}\right] \tag{10}$$

where M_j, ζ_j and α_j are the neutralizer's mass, damping ratio and tuning ratio respectively. α_j is equal to ω/ω_j where $\omega_j = (k_j/M_j)^{1/2}$ is the neutralizer's natural frequency and k_j is the stiffness of the j-th neutralizer. M_j and ζ_j are determined using the numerical approach described in [11] and [12]. Taking advantage of the quadratic minimization method described in Section 2.1, Eq. (9) can be combined with Eq. (7) to give the required dynamic stiffness of the j-th neutralizer as

$$K_{r(\mathrm{opt})j} = -f_{s(\mathrm{opt})j}\left(\boldsymbol{\Phi}^T(x_j)\mathbf{q}_{(\mathrm{opt})}\right)^{-1} \tag{11}$$

where $f_{s(\mathrm{opt})j}$ is the optimum value of the j-th secondary force calculated using Eq. (7). Eq. (11) is the dynamic stiffness required to produce the same reduction in the kinetic energy of the host structure as an active device does. It has an imaginary and real component as discussed in [10]. The optimum tuning ratio of the j-th neutralizer is then given by

$$\alpha_{j(\mathrm{opt})} = \sqrt{1 - \frac{M_j \omega^2}{\mathrm{Re}\left[K_{\mathrm{r(opt)}j}\right]}} \tag{12}$$

where $\mathrm{Re}\left[K_{\mathrm{r(opt)}j}\right]$ is the real part of Eq. (11). Substituting for α_j from Eq. (12) into Eq. (10) gives the optimal dynamic stiffness of the neutralizer, and further substitution into Eq. (9) gives the optimal feedback forces from neutralizers, equivalent to the optimal secondary forces given in Eq. (7).

3. Experimental setup and measurements

3.1. The host structure and the neutralizer

To simplify the experimental work, a cantilever beam was chosen to be the host structure. One of the reasons of this selection was because it is relatively easy to set-up the experimental rig without losing generality. Fig. 2 shows the combined system used in this experiment, which comprises of a cantilever as a host structure and a neutralizer as a control device. Fig. 2(a) shows the plan view of the combined system, Fig. 2(b) shows its side view and Fig. 2(c) shows the equivalent diagram of the combined system. The neutralizer used in this experiment was a free-free beam and was made of aluminum, bolted at its mid point onto the cantilever, which makes a **double cantilever neutralizer**. To ensure a point feedback force from the neutralizer, a small, thin washer was placed between the neutralizer and the host structure [Fig. 2(b)].

The double cantilever neutralizer can be modeled as a two-degree of freedom system consisting of two masses, M_{e} and M_{at}, connected by a spring and a damper as shown in Fig. 2(c). M_{e} is the effective mass of the neutralizer which is 59.6% of the total mass of the free-free beam, and M_{at} is a mass which is 40.4% of the free-free beam, which is effectively added onto the host structure when the neutralizer is fitted [16]. Therefore, the natural frequency of the neutralizer is given by

$$\omega_{\mathrm{a}} = \sqrt{\frac{k_{\mathrm{e}}}{M_{\mathrm{e}}}} \tag{13}$$

where k_{e} is the equivalent stiffness of the free-free beam. The natural frequency of the neutralizer in Eq. (13) is the same as the natural frequency of a cantilever beam with the length of half of the free-free beam, which is $L_a/2$. This is the frequency at which the neutralizer causes the highest reduction in the frequency response of the host structure.

The cantilever beam used as a host structure in this experiment was made of steel with the dimensions of $0.5\times0.025\times0.006$ m, and the frequency range of interest was between 300 and 500 Hz. A point primary force was applied at $x_f = 0.025$ m and the

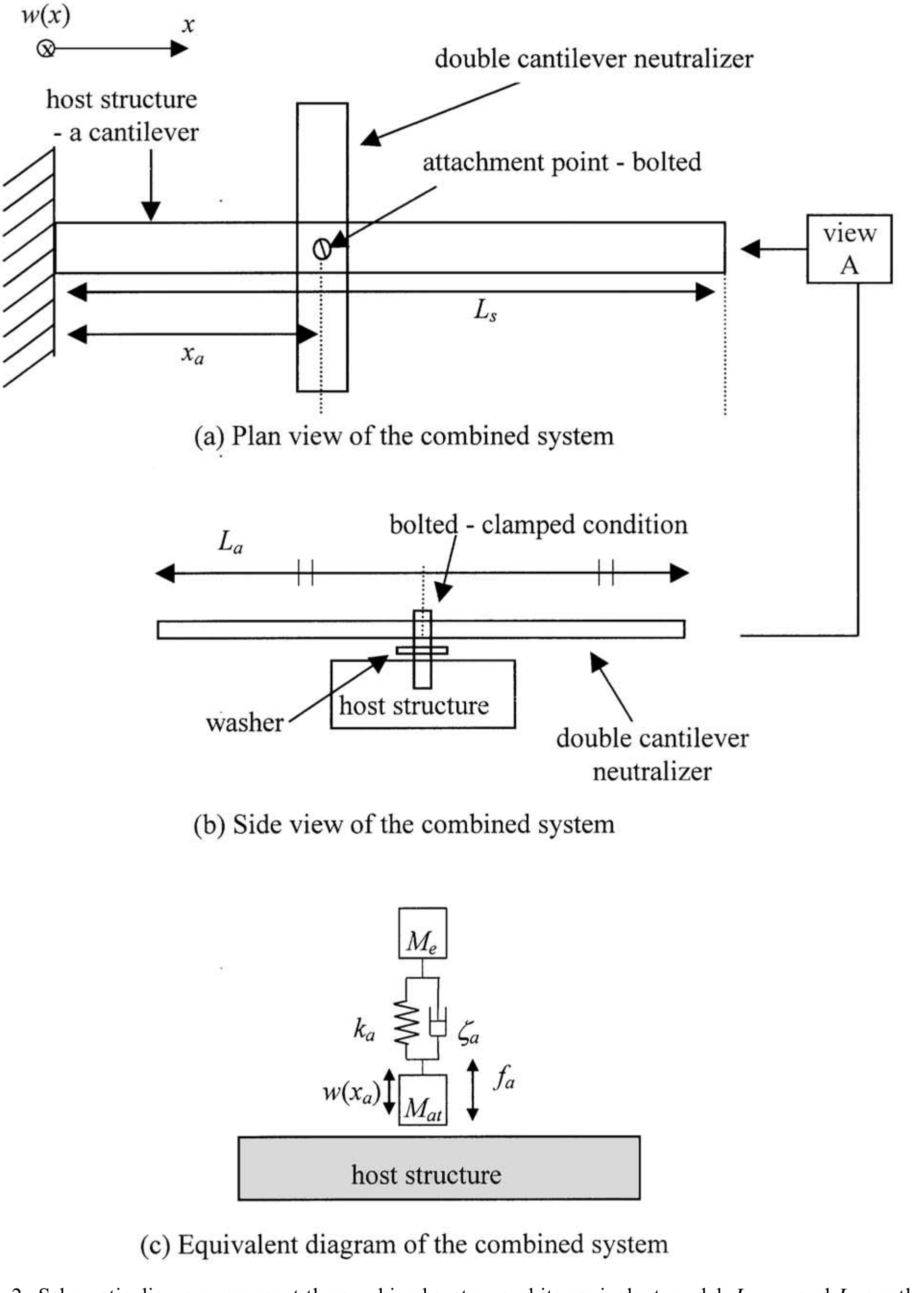


Fig. 2. Schematic diagram represent the combined system and its equivalent model. L_s, x_a, and L_a are the length of the cantilever, the location of the neutralizer on the cantilever and the length of the double cantilevers neutralizer respectively. ⊗ means the axis is pointing out of the page.

modal damping ratio of the cantilever was set to be 0.05. This value was determined by comparing the magnitude of the measured frequency response of the cantilever with its theoretical prediction at its natural frequencies.

3.2. Physical properties of the neutralizers

The objective of this experiment was to show that a passive control device such as a neutralizer could be as effective as an active device in reducing the global response of a structure when using the proposed optimal tuning method in Ref. [10]. In order to achieve this, a simple approach was adopted by using six free-free beams as neutralizers to control the kinetic energy of the host structure at six different target frequencies within the specified range of interest. This was to simulate the result if a single optimally detuned tunable vibration neutralizer was used. Three conditions were investigated in this experiment [10].

a. When both real and imaginary parts of the required dynamic stiffness of Eq. (11) are positive, that is when the control device is required to be stiffness-like and to absorb energy from the host structure.
b. When the real part is negative but the imaginary part of the required dynamic stiffness is positive, that is when the control device is required to be a mass-like and to absorb energy from the host structure.
c. When both real and imaginary parts of the required dynamic stiffness are negative, that is when the control device is required to be a mass-like and to supply energy to the host structure.

It is of particular interest to investigate condition (c) when the control device is required to supply energy to the host structure.

The relevant frequencies are 337 and 354 Hz for condition 1,376 and 393 Hz for condition 2, and 429 and 439 Hz for condition 3; the neutralizers were to be applied at $x_a = 0.198$ m one at a time. These frequencies were selected by examining the simulation results of the real and imaginary parts of the required dynamic stiffness of the neutralizer that fulfill the above conditions, and are summarized in Table 1. For the target frequency number 3 (i.e. 376 Hz) in Table 1, the optimal frequency of the neutralizer is 375.624 Hz but it is rounded to 376 Hz.

The optimal frequency of each of the double cantilever neutralizer, $\omega_{a(opt)}$ described in Table 1 was determined using the following equation [10].

$$\omega_{a(opt)} = \frac{\omega}{\alpha_{a(opt)}} \tag{14}$$

ω and $\alpha_{a(opt)}$ are the forcing frequency and the optimal tuning ratio of the neutralizer respectively. The optimal tuning ratio $\alpha_{a(opt)}$ was determined using Eq. (12). Note that the index of the neutralizer is now changed to $_a$ because there is effectively only one neutralizer used in the whole frequency range of interest. μ_a/ζ_a of the neutralizer is fixed at 10 with $\zeta_a = 0.001$. This is the optimal value of μ_a/ζ_a operates at the third

Table 1
Selected target frequencies, the condition it represents, the optimal tuning ratio and the optimal natural frequency of the neutralize

No.	Target frequency (Hz)	Real part of required dynamic stiffness	Imaginary part of the required dynamic stiffness	$\alpha_{a(opt)}$	Optimal frequency (Hz)[a]
1	337	Positive—stiffness- like passive system	Positive—the control device is required to absorb energy	1.0051	335
2	354			1.0026	353
3	376	Negative—mass-like passive system	Positive—the control device is required to absorb energy	0.9990	376
4	393			0.9960	395
5	429	Negative—mass-like passive system	Negative—the control device is required to supply energy	0.9880	434
6	439			0.9520	446

[a] Rounded natural frequency of the neutralizer. The mass and damping ratios of the neutralizer are fixed at $\mu_a = 0.0138$ and $\zeta_a = 0.001$ respectively, and the neutralizer is applied at $x_a = 0.198$ m. The primary force is applied at $x_f = 0.025$ m.

natural frequency of the host cantilever as determined using the numerical procedure described in [11,12]. The damping ratios of the neutralizers (ζ_a) were determined by comparing the measured frequency response of the neutralizer with the theoretical prediction at its first resonance frequency, and the natural frequency of the neutralizer was determined by measuring the accelerance at the center of the double cantilevers neutralizer.

The physical properties, the dimensions, the mass and the natural frequencies of the neutralizers used in this experiment are summarized in Table 2. The natural frequency of the neutralizer is the first anti-resonance of the frequency response function [2].

Referring to Eq. (10), the optimal dynamic stiffness of the neutralizer can be written as

Table 2
Physical properties of the neutralizers. Young's modulus = 71E9 N/m^2, $\rho = 2770$ kg/m^3, $\zeta_a = 0.001$, thickness = 2 mm

No. of neutralizer	L_a (mm)	Width (mm)	Effective mass, M_e (kg)	Measured natural frequency (Hz)
1	142	18	0.0084	335
2	138	18	0.0082	353
3	134	19	0.0084	376
4	130.6	19	0.0082	395
5	124.8	20	0.0082	434
6	123.2	21	0.0085	446

$$K_{a(opt)} = -M_e\omega^2\left[\frac{1 + i2\zeta_a\alpha_{a(opt)}}{1 - \alpha^2_{a(opt)} + i2\zeta_a\alpha_{a(opt)}}\right] \tag{15}$$

where ζ_a is the damping ratio of the neutralizer, and $\alpha_{a(opt)}$ is the optimal tuning ratio between the forcing frequency ω to the natural frequency of the neutralizer [Eq. (14)], ω_a. Combining Eqs. (15), (9), (8) and (6) gives the kinetic energy of the cantilever. It should be noted that the effective added mass M_{at} onto the mass of the host structure was ignored because it is very small (around 0.006 kg) compared to the total mass of the cantilever, which is around 0.6 kg.

3.3. Measurements

The neutralizers were attached to the host structure (refer to Fig. 2) at $x_a = 0.198$ m one at a time to determine the effect on the kinetic energy of the cantilever at their respective target frequencies. The host structure was virtually divided into ten subsections with equal length of 0.05 meter (Fig. 3). With the first neutralizer attached, the cantilever was excited at the center point of its first subsection near to the clamped end ($x_f = 0.025$ m) using an impact hammer. The response of the cantilever was measured at the center point of each subsection using an accelerometer. The signals from the hammer and the accelerometer were conditioned using charge amplifiers and fed into a frequency analyzer to give the accelerance transfer function. Since the measured transfer functions were in terms of acceleration, dividing

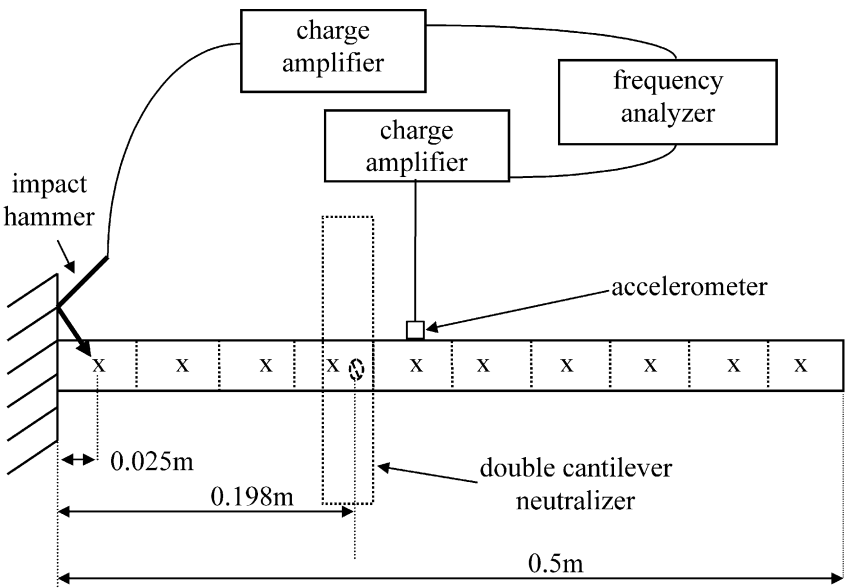


Fig. 3. Experimental set-up to determine the effect of the application of an optimally detuned neutralizer to the kinetic energy of the cantilever. X—location of the accelerometer.

the measured transfer functions by $i\omega$ gave the function in term of velocity. In mathematical terms, if e_j is the measured acceleration transfer function from the j-th subsection in the modal domain, then the kinetic energy of the j-th subsection is given by

$$KE_j = \frac{\rho w h L_s}{40} \left| \frac{e_j}{i\omega} \right|^2 \tag{16}$$

where KE_j is the kinetic energy of the j-th subsection and ρ, w, h and L_s are the material density, the width, the thickness and the total length of the cantilever respectively. Therefore, the total kinetic energy of the cantilever with 10 equal subsections is the summation of the kinetic energy of each subsection, which is

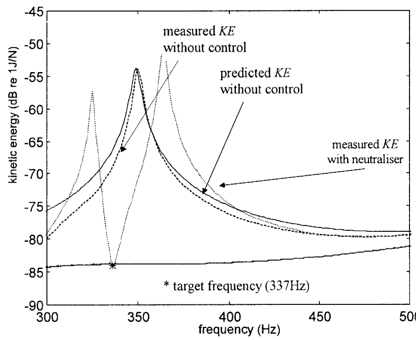
$$KE = \sum_{j=1}^{J=10} KE_j \tag{17}$$

The same procedure was followed for all of the neutralizers to determine their effects on the kinetic energy of the cantilever at their respective target frequency.

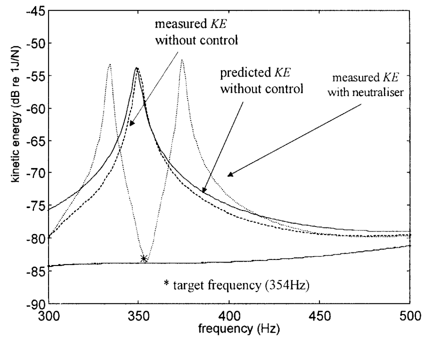
4. Results and discussion

Fig. 4 shows the predicted and measured kinetic energy of the host structure without control, and the measured kinetic energy when the six optimally detuned neutralizers were attached at $x_a = 0.198$ m one at a time. Also plotted is the predicted kinetic energy of the host structure when a single optimally detuned neutralizer (predicted optimal passive), and when an active device is attached. M_e for predicted optimal passive was fixed at 0.0082 kg. It should be noted that the predicted kinetic energy with an active control device fitted is used as a benchmark to evaluate the performance of the neutralizer in reducing the kinetic energy of the host structure. The *s in the figures are the measured kinetic energy of the cantilever at the target frequencies. It can be seen that at these target frequencies, the kinetic energy is minimized when the appropriate natural frequency of the neutralizer is selected, and this agrees with the theoretical prediction from the suggested optimum tuning method and also when using active control.

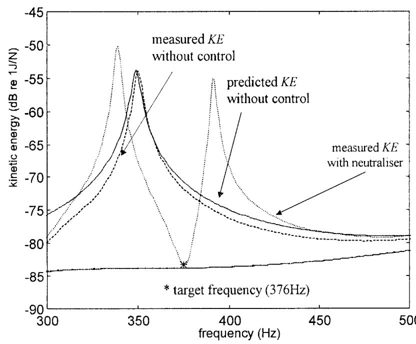
Fig. 4(e) and (f) are particularly of interest where the control device is required to supply energy to the host structure. The kinetic energy of the cantilever at the targeted frequencies 429 and 439 Hz is reduced by 4 and 3dB respectively, compared to the measured kinetic energy without any control device fitted. These attenuations are slightly lower than those predicted or when an active device is used. Although there are small differences compared to the predictions, the results are in reasonable agreement. This shows that a passive device such as a neutralizer, which has no ability to supply energy to its host structure, can be as effective as an active device if it is properly optimized.



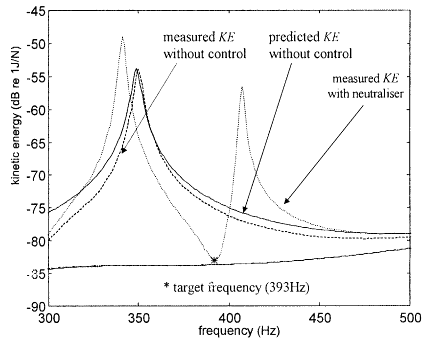
(a) Neutralizer no. 1 with ω_a=335Hz



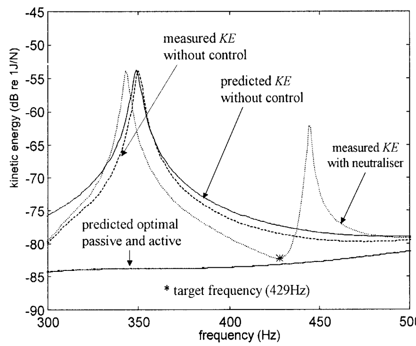
(b) Neutralizer no. 2 with ω_a=353Hz



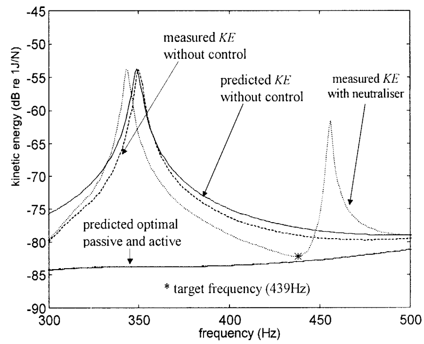
(c) Neutralizer no. 3 with ω_a=376Hz



(d) Neutralizer no. 4 with ω_a=395Hz



(e) Neutralizer no. 5 with ω_a=434Hz



(f) Neutralizer no. 6 with ω_a=446Hz

Fig. 4. Measured kinetic energy of the host structure without control and with each neutralizer fitted at $x_a = 0.198$ m, compared to the predicted kinetic energy without control, with each neutralizer fitted, with single optimally detuned neutralizer (optimal passive) and with active control as a benchmark.

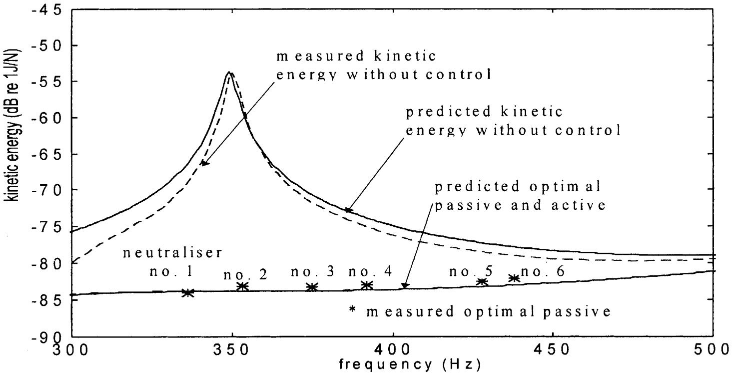


Fig. 5. Experimental results on the kinetic energy of the cantilever using neutralizer with optimal tuning ratio at the targeted frequencies. The *s are the measured kinetic energy at the target frequencies of the neutralizer with the optimal frequency attached. M_e for predicted optimal passive is fixed at 0.0082 kg and the result from active control is used as a benchmark.

Fig. 5 summarizes the results presented in Fig. 4; that is the kinetic energy of the cantilever at the target frequencies for all six optimally detuned neutralizers placed one at a time. This figure can be considered as the kinetic energy of the cantilever beam when only one optimally detuned tunable vibration neutralizer is in used. Overall, the kinetic energy of the cantilever at the target frequencies when each of the neutralizers was applied is within ± 1 dB of the predicted kinetic energy when either a single optimally detuned neutralizer or an active device is used.

5. Conclusions

Experimental evidence to support the proposed optimal tuning of a vibration neutralizer in reducing the global vibration of a continuous structure has been presented in this paper. It has been shown that the effectiveness of the vibration neutralizer is comparable to that of active control as predicted by the authors [10].

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