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Derivation of the Fixed-Points Theory with Some Numerical Simulations for Global Vibration Control of Structure with Closely Spaced Natural Frequencies[#]

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Abstract: The fixed-points theory has been used as one of the design laws in fabricating a vibration neutralizer for the control of a relatively simple structure. The underlying principle of the theory is that in the frequency response function (FRF) of the system considered, there exist two fixed points that are common to all FRF curves regardless of the damping value of the neutralizer. It is possible, with the proper selection of the neutralizer's resonance frequency, to determine the optimal damping value of the neutralizer that provides a smooth FRF by following the standard procedure of the theory. Recently, the authors have extended the application of the theory for global vibration control of a continuous structure with well separated natural frequencies. In this paper, the application is further extended to global vibration control of a structure with natural frequencies that are closely spaced. Through some numerical simulations, it is shown that the theory can also be used to remove the effects of the dominant mode within the frequency range where the global response is

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dominated by that mode. However, there are some limitations in its application especially for the overlapping natural frequencies.

Keywords: Fixed-points theory; Flexible structures; Global vibration control; Passive vibration control; Vibration adsorbers; Vibration neutralizers.

INTRODUCTION

The fixed-points theory dates back to 1932 when Hahnkamm (1932) suggested that there are two fixed points in the primary structure's frequency response function (FRF) of two degree of freedom system when a harmonic force is applied to the primary mass. These points are independent of the damping value in the auxiliary system, which is the control system, and their heights are mainly determined by the mass ratio of the device. The desired optimal value of the tuning ratio is obtained when the height of the fixed points are equal. Fourteen years later, Brock (1946) suggested the optimum value of the damping ratio in the control device could be determined by making the height of the fixed points the maximum. The theory has been well documented in the textbook by den Hartog (1956). Since then, the fixed-points theory has been successfully used in many applications as one of the design laws in fabricating a vibration neutralizer (Ren, 2001).

Unfortunately, these applications are confined only to the control of a relatively simple structure or to control the point response of a continuous structure with well separated natural frequencies such as beams (Stefen and Rade, 2002). However, for a continuous structure, reducing the vibration amplitudes at a point may increase its amplitudes at other points (Dayou, 1999). In order to achieve overall reduction, global measures of the continuous structure must be chosen as a cost function to be minimized. It has been shown that these two fixed points also exist in the kinetic energy (as a measure of the global behavior) of a beam (Dayou and Wang, 2004). As a consequence, it was then proven that the theory can be used to remove the effects of the dominant mode leaving only the effect from residual modes in the global response of the beam (Dayou, 2005, 2006).

In this paper, the application of the fixed-points theory is further developed and examined for the control of global vibration of two dimensional structure such as plates. The structure has natural frequencies that are close to each other and its kinetic energy is chosen as a cost function to be minimized. Through some numerical simulations, it is shown that the theory can also be used to remove the effect of the dominant mode in the frequency range of interest, as in the case of the beam.

PROBLEM FORMULATION

The transverse displacement of the plate w at any point on its two-dimensional surface (x, y) can be written in terms of finite mode MN as

$$w(x, y) = \mathbf{\Phi}^T\mathbf{q} \tag{1}$$

where $w(x, y)$ is the displacement at the location (x, y), $\mathbf{\Phi}$ is the vector of the mnth normalized mode shape of the plate evaluated at (x, y), and $\mathbf{q}$ is the vector of the modal displacement amplitude. The vector of the modal displacement amplitudes is given by $\mathbf{q} = \mathbf{Ag}$ where $\mathbf{A}$ is the complex modal displacement amplitude whose elements are given by

$$A_{mn} = \frac{1}{M_s(\omega_{mn}^2 - \omega^2 + i2\zeta_{mn}\omega_{mn}\omega)} \tag{2}$$

and $\mathbf{g}$ is the vector of the generalized force acting on the structure. In Eq. (2), M_s is the modal mass, ζ_{mn} is the modal damping ratio, ω is the circular frequency of the primary force, ω_{mn} is the mnth circular natural frequency of the structure, and i is the imaginary number given by $\sqrt{-1}$.

If a vibration neutralizer is used as a control device and is fitted on the plate, then there are two contributing forces that make up the generalized force, $\mathbf{g}$—the primary uncontrolled force and the feedback force generated by the neutralizer. Therefore, the modal displacement amplitude is written as

$$\mathbf{q} = \mathbf{A}(\mathbf{g}_p + \mathbf{\Phi}_k \mathrm{f}_k) \tag{3}$$

where $\mathbf{g}_p$, $\mathbf{\Phi}_k$, and f_k are the vector of the generalized primary force, the normalized mode shape of the plate evaluated at the neutralizer's location, and the amplitude of the feedback force from the neutralizer, respectively. Note that $\mathbf{\Phi}_k \mathrm{f}_k$ is the generalized feedback force from the vibration neutralizer.

The feedback force from the neutralizer can be written as (Jones, 1967)

$$\mathrm{f}_k = -K_k w(x, y) \tag{4}$$

where K_k is the dynamic stiffness of the neutralizer given by

$$K_k = -\omega^2 M_k \left[\frac{1 + i2\zeta_k(\omega/\omega_k)}{1 - (\omega/\omega_k)^2 + i2\zeta_k(\omega/\omega_k)}\right]. \tag{5}$$

M_k, ζ_k, and ω_k are the mass, the damping ratio, and the resonance frequency of the neutralizer, respectively. The damping ratio of the neutralizer is defined as $\zeta_k = C_{ck}/(2M_k\omega_k)$ where C_{ck} is the neutralizer's critical damping, whereas the resonance frequency is $\omega_k = \sqrt{k_k/M_k}$, and k_k is the neutralizer's stiffness constant. Substituting Eq. (1) into (4) gives the neutralizer's feedback force in terms of the modal amplitude of the structure as

$$\mathrm{f}_k = -K_k \mathbf{\Phi}^T \mathbf{q}. \tag{6}$$

Therefore, combining Eqs. (3) and (6) gives the modal displacement amplitude as

$$\mathbf{q} = \mathbf{A}(\mathbf{g}_p - K_k \mathbf{\Phi}_k^T \mathbf{\Phi}_k \mathbf{q}) \tag{7}$$

or

$$\mathbf{q} = [\mathbf{I} + K_k \mathbf{A} \mathbf{\Phi}_k^T \mathbf{\Phi}_k]^{-1} \mathbf{A} \mathbf{g}_p. \tag{8}$$

Combination of Eqs. (1) and (8) represents the behavior of the plate at a particular point with a neutralizer attached and this can be used to evaluate the neutralizer's performance for the purpose of point control. However, it is well known that reducing the vibration amplitude at a particular point may increase the amplitude at some other points (Dayou, 1999). Therefore, a global term is required to evaluate the total effectiveness of the control device. In this paper, the time averaged kinetic energy is used as a measure of the global behavior of the system. The time averaged kinetic energy (or simply kinetic energy) is given by (Nelson and Elliott, 1992)

$$KE = \frac{M_s \omega^2}{4} \mathbf{q}^H \mathbf{q} \tag{9}$$

where the superscript H denotes the Hermitian transpose.

OPTIMUM TUNING AND DAMPING RATIOS OF THE NEUTRALIZER

As described earlier, the fixed-points theory was originally developed for the control of a relatively simple structure. The theory was then developed for point response control of a continuous structure with well separated natural frequencies (Stefen and Rade, 2002). Later, the theory was developed for the control of global behavior of structure of well separated natural frequencies, in the vicinity of the natural frequency of interest (Dayou, 2005, 2006). In this paper, the application of the theory is further developed and examined to be used in the global vibration control of a structure that has natural frequencies that are close to each other. However, the question on the applicability of the theory to such a problem will not be dealt with at this stage. The theory is simply developed in this section and the question of the possibility of application is addressed in a later section.

At this stage, let us simply assume that the modal displacement amplitude in the vicinity of mnth natural frequency of the structure can be approximated by its mnth mode only. In this situation, Eq. (8) can be rewritten as

$$q_{mn} = \frac{A_{mn} g_{pmn}}{1 + K_k A_{mn} \phi_{mn}^2(x_k, y_k)}. \tag{10}$$

Let us also assume that there is no damping in the structure. Therefore, the complex modal amplitude of the plate can be rearranged as

$$A_{mn} = \frac{1}{(k_{mn} - M_s \omega^2)} \tag{11}$$

whereas the dynamic stiffness of the neutralizer is rewritten as

$$K_k = -M_k \omega^2 \left[\frac{k_k + i2\zeta_k \sqrt{M_k k_k}\,\omega}{k_k - M_k \omega^2 + i2\zeta_k \sqrt{M_k k_k}\,\omega} \right]. \tag{12}$$

k_{mn} in Eq. (11) can be regarded as the mnth effective bending stiffness of the plate given by

$$k_{mn} = \left[\left(\frac{m\pi}{L_x} \right)^2 + \left(\frac{n\pi}{L_y} \right)^2 \right]^2 \frac{E h^3 L_x L_y}{1 - v^2}. \tag{13}$$

E, h, L_x, L_y, and v are the Young's modulus, the thickness, the length, and the width of the plate, respectively. By using Eqs. (11) and (12), Eq. (10) can be expressed as

$$q_{mn} = \frac{g_{pmn}\left[(i2\zeta_k \sqrt{M_k k_k}\,\omega) + \{k_k - M_k \omega^2\} \right]}{\begin{array}{r}\left[(i2\zeta_k \sqrt{M_k k_k}\,\omega)\{k_{mn} - M_s \omega^2 - M_k \omega^2 \phi_{mn}^2(x_k, y_k)\} \right. \\ \left. + \{(k_{mn} - M_s \omega^2)(k_k - M_k \omega^2) - M_k k_k \omega^2 \phi_{mn}^2(x_k, y_k)\} \right]\end{array}}. \tag{14}$$

Therefore, the kinetic energy of the plate in the vicinity of its mnth natural frequency can be written as

$$KE_{mn} = \frac{M_s \omega^2 g_{pmn}^2}{4} \times \frac{\left[(2\zeta_k \sqrt{M_k k_k}\,\omega)^2 + \{k_k - M_k \omega^2\}^2 \right]}{\begin{array}{r}\left[(2\zeta_k \sqrt{M_k k_k}\,\omega)^2 \{k_{mn} - M_s \omega^2 - M_k \omega^2 \phi_{mn}^2(x_k, y_k)\}^2 \right. \\ \left. + \{(k_{mn} - M_s \omega^2)(k_k - M_k \omega^2) - M_k k_k \omega^2 \phi_{mn}^2(x_k, y_k)\}^2 \right]\end{array}}. \tag{15}$$

If the equation is divided by $(M_k/M_k)^2$ then

$$KE_{mn} = \frac{M_s\omega^2 g_{pmn}^2}{4} \times \frac{\left[(2\zeta_k\omega_k\omega)^2 + \{\omega_k^2 - \omega^2\}^2\right]}{\left[(2\zeta_k\omega_k\omega)^2\{k_{mn} - M_s\omega^2 - M_k\omega^2\phi_{mn}^2(x_k, y_k)\}^2 + \{(k_{mn} - M_s\omega^2)(\omega_k^2 - \omega^2) - M_k\omega_k^2\omega^2\phi_{mn}^2(x_k, y_k)\}^2\right]}. \tag{16}$$

Equation (16) can also be written as

$$KE_{mn} = \frac{\omega^2 g_{pmn}^2}{4M_s} \frac{\left[(2\zeta_k\omega_k\omega)^2 + \{\omega_k^2 - \omega^2\}^2\right]}{\left[(2\zeta_k\omega_k\omega)^2\{\omega_{mn}^2 - \omega^2 - \mu\omega^2\phi_{mn}^2(x_k, y_k)\}^2 + \{(\omega_{mn}^2 - \omega^2)(\omega_k^2 - \omega^2) - \mu\omega_k^2\omega^2\phi_{mn}^2(x_k, y_k)\}^2\right]} \tag{17}$$

where μ is the mass ratio given by M_k/M_s. Equation (17) can be divided by $(\omega_{mn}/\omega_{mn})^4$ to get

$$KE_{mn} = \frac{\omega^2 g_{pmn}^2}{4M_s} \frac{\left[(2\zeta_k f_{mn} g_{mn})^2 + \{f_{mn}^2 - g_{mn}^2\}^2\right]}{\left[(2\zeta_k f_{mn} g_{mn})^2\{\omega_{mn}^2 - \omega^2 - \mu\omega^2\phi_{mn}^2(x_k, y_k)\}^2 + \{(1 - g_{mn}^2)(\omega_k^2 - \omega^2) - \mu f_{mn}^2\omega^2\phi_{mn}^2(x_k, y_k)\}^2\right]} \tag{18}$$

where

$$\begin{aligned} f_{mn} &= \omega_k/\omega_{mn} \\ g_{mn} &= \omega/\omega_{mn}. \end{aligned} \tag{19}$$

Dividing only the denominator by $\omega_{mn}^4/\omega_{mn}^4$ (where $\omega_{mn} = \sqrt{k_{mn}/M_s}$) yields

$$KE_{mn} = \frac{M_s\omega^2 g_{pmn}^2}{4k_{mn}^2} \frac{\left[(2\zeta_k f_{mn} g_{mn})^2 + \{f_{mn}^2 - g_{mn}^2\}^2\right]}{\left[(2\zeta_k f_{mn} g_{mn})^2\{1 - g_{mn}^2 - \mu g_{mn}^2\phi_{mn}^2(x_k, y_k)\}^2 + \{(1 - g_{mn}^2)(f_{mn}^2 - g_{mn}^2) - \mu f_{mn}^2 g_{mn}^2\phi_{mn}^2(x_k, y_k)\}^2\right]}. \tag{20}$$

Equation (20) is the kinetic energy of the *mn*th natural frequency of the plate in its dimensionless form. This can be rearranged to give

$$KE_{mn} = \frac{M_s\omega^2 g_{pmn}^2}{4k_{mn}^2}\left(\frac{A^2\zeta_k^2 + B^2}{C^2\zeta_k^2 + D^2}\right) \tag{21}$$

where

$$\begin{aligned} A &= 2f_{mn}g_{mn}, \\ B &= g_{mn}^2 - f_{mn}^2, \\ C &= 2f_{mn}g_{mn}\{g_{mn}^2 - 1 + \mu g_{mn}^2\phi_{mn}^2(x_k, y_k)\}, \\ D &= \mu f_{mn}^2 g_{mn}^2 \phi_{mn}^2(x_k, y_k) - (g_{mn}^2 - 1)(g_{mn}^2 - f_{mn}^2). \end{aligned} \tag{22}$$

Equation (21) has the same form as for the simple primary system described by Brock (1946) and also by den den Hartog (1956), which makes it possible to define two invariant points regardless of the damping value in the vibration neutralizer when $A/C = B/D$.

Following the classical fixed-points theory, the two fixed points can be established by considering two cases of kinetic energy of the structure: when the neutralizer's damping ratio is zero and when it is infinity. Using Eq. (21), these two cases can be expressed respectively as

$$\gamma_{mn}|_{\zeta_k=0} = \left(\frac{B^2}{D^2}\right) \tag{23}$$

$$\gamma_{mn}|_{\zeta_k=\infty} = \left(\frac{A^2}{C^2}\right), \tag{24}$$

where

$$\gamma_{mn} = KE_{mn} \Big/ \frac{M_s \omega^2 g_{pmn}^2}{4k_{mn}^2}. \tag{25}$$

The condition $(B/D)^2 = (A/C)^2$ implies the two crossing points of curves $\gamma_{mn}|_{\zeta_k=0}$ and $\gamma_{mn}|_{\zeta_k=\infty}$. By using this crossing point condition, based on Eq. (22), it can be written that

$$\left\{\frac{g_{mn}^2 - f_{mn}^2}{\mu f_{mn}^2 g_{mn}^2 \phi_{mn}^2(x_k, y_k) - (g_{mn}^2 - 1)(g_{mn}^2 - f_{mn}^2)}\right\}^2 = \left\{\frac{1}{g_{mn}^2 - 1 + \mu g_{mn}^2 \phi_{mn}^2(x_k, y_k)}\right\}^2. \tag{26}$$

Equation (26) can be reduced to a simpler form by taking its square roots but a $\pm$ve sign must be added to the right hand side of the equation. Equation with a $-$ve sign is the trivial solution, therefore the fixed-points equation is given by

$$\frac{g_{mn}^2 - f_{mn}^2}{\mu f_{mn}^2 g_{mn}^2 \phi_{mn}^2(x_k, y_k) - (g_{mn}^2 - 1)(g_{mn}^2 - f_{mn}^2)} = \frac{1}{g_{mn}^2 - 1 + \mu g_{mn}^2 \phi_{mn}^2(x_k, y_k)}. \tag{27}$$

Cross-multiplication and rearrangement yields

$$g_{mn}^4 - 2g_{mn}^2\left\{\frac{1 + f_{mn}^2 + \mu f_{mn}^2\phi_{mn}^2(x_k, y_k)}{2 + \mu\phi_{mn}^2(x_k, y_k)}\right\} + f_{mn}^2\left\{\frac{2}{2 + \mu\phi_{mn}^2(x_k, y_k)}\right\} = 0. \tag{28}$$

Suppose g_{mn1}^2 and g_{mn2}^2 are the roots of this equation, then

$$(g_{mn}^2 - g_{mn1}^2)(g_{mn}^2 - g_{mn2}^2) = g_{mn}^4 - (g_{mn1}^2 + g_{mn2}^2)g_{mn}^2 + g_{mn1}^2 g_{mn2}^2 = 0. \tag{29}$$

Again, by comparing Eqs. (28) and (29), one obtains

$$g_{mn1}^2 + g_{mn2}^2 = 2\left\{\frac{1 + f_{mn}^2 + \mu f_{mn}^2\phi_{mn}^2(x_k, y_k)}{2 + \mu\phi_{mn}^2(x_k, y_k)}\right\}. \tag{30}$$

According to the fixed-points theory, the kinetic energy at these two roots must be equal regardless of the damping in the neutralizer. This occurs when either Eq. (23) or (24) is satisfied. For simplification, Eq. (24) is used and substituting the two roots,

$$\gamma_{mn}\big|_{\zeta_k=\infty} = \left\{\frac{1}{g_{mn1}g_{mn2}(1 + \mu\phi_{mn}^2(x_k, y_k)) - 1}\right\}^2 \tag{31}$$

or

$$\sqrt{\gamma_m\big|_{\zeta_k=\infty}} = \pm\frac{1}{g_{m1}g_{m2}(1 + \mu\phi_{mn}^2(x_k, y_k)) - 1}. \tag{32}$$

However, the two equations in Eq. (32) must be the same according to the fixed-points theory and therefore

$$\frac{1}{g_{mn1}^2(1 + \mu\phi_{mn}^2(x_k, y_k)) - 1} = \frac{-1}{g_{mn2}^2(1 + \mu\phi_{mn}^2(x_k, y_k)) - 1} \tag{33}$$

or

$$g_{mn1}^2 + g_{mn2}^2 = \frac{2}{1 + \mu\phi_{mn}^2(x_k, y_k)}. \tag{34}$$

Comparing Eqs. (30) and (34), the optimum tuning condition, which is the desirable tuning ratio, is obtained when

$$f_{mn\,\mathrm{opt}} = \frac{1}{1 + \mu\phi_{mn}^2(x_k, y_k)} \tag{35}$$

This has a similar form with the optimum tuning condition for the simple primary system given by den Hartog (1956), with an additional term which is the modal amplitude of the structure at the neutralizer's location, $\phi^2_{mn}(x_k, y_k)$. This implies the importance of the positioning of the control device on the structure.

Substituting Eq. (35) into (28) gives the abscissas of the fixed points in $(g, |\gamma_{m,n}|)$ diagram as

$$g_{1,2} = \sqrt{\frac{1 \pm \sqrt{\frac{\mu\phi^2_{mn}(x_k,y_k)}{2+\mu\phi^2_{mn}(x_k,y_k)}}}{1 + \mu\phi^2_{mn}(x_k, y_k)}} \tag{36}$$

while the common ordinate (the height of the fixed points) is

$$|\gamma_{mn}| = 1 + \frac{2}{\mu\phi^2_{mn}(x_k, y_k)}, \tag{37}$$

which can be derived by substituting Eq. (36) into (32).

The next task is to derive the optimum damping of the vibration neutralizer. From Eq. (21), the neutralizer's damping ratio can be written as

$$\zeta_k^2 = \frac{\left[g^2_{mn} - f^2_{mn}\right]^2 - \gamma_{mn}\left[\mu\phi^2_{mn}(x_k, y_k) f^2_{mn} g^2_{mn} - (g^2_{mn} - 1)(g^2_{mn} - f^2_{mn})\right]^2}{4 f^2_{mn} g^2_{mn}\left[\gamma_{mn}(g^2_{mn} - 1 + \mu\phi^2_{mn}(x_k, y_k) g^2_{mn})^2 - 1\right]}. \tag{38}$$

Suppose the two fixed points on the nondimensional kinetic energy curve are P and Q. In order for the curve to pass horizontally through the first fixed point P, it is required that it passes through a point P' of the abscissa

$$g_1 = \sqrt{\frac{1 - \sqrt{\frac{\mu\phi^2_{mn}(x_k,y_k)}{2+\mu\phi^2_{mn}(x_k,y_k)}} + \delta}{1 + \mu\phi^2_{mn}(x_k, y_k)}} \tag{39}$$

with the ordinate given in Eq. (37). If δ approaches zero as a limit, substituting Eqs. (35), (37), and (39) into (38) will give result in the form of

$$\zeta_k^2 = \frac{(A_o + A_1\delta + A_2\delta^2 + A_3\delta^3 + \cdots)}{(B_o + B_1\delta + B_2\delta^2 + B_3\delta^3 + \cdots)}. \tag{40}$$

If $\delta = 0$, then the nondimensional kinetic energy curve lies on the fixed point and ζ_k is assumed to be indeterminate because it can take infinite number of values. Therefore, $A_o = B_o$. If δ is not zero but has a very small value, other terms in Eq. (40) that were multiplied with δ of power higher than unity can be neglected, leaving only

$$\zeta_k^2 = \frac{A_1}{B_1}. \tag{41}$$

Therefore, substituting Eqs. (35), (37), and (39) into (38), and taking only the terms that were multiplied with δ gives, after rearrangement

$$\zeta_{k1}^2 = \frac{\mu\phi_{mn}^2(x_k, y_k)\left[3 - \sqrt{\frac{\mu\phi_{mn}^2(x_k,y_k)}{2+\mu\phi_{mn}^2(x_k,y_k)}}\right]}{8(1+\mu\phi_{mn}^2(x_k, y_k))}. \tag{42}$$

Following a similar procedure for g_2, one will obtain

$$\zeta_{k2}^2 = \frac{\mu\phi_{mn}^2(x_k, y_k)\left[3 + \sqrt{\frac{\mu\phi_{mn}^2(x_k,y_k)}{2+\mu\phi_{mn}^2(x_k,y_k)}}\right]}{8(1+\mu\phi_{mn}^2(x_k, y_k))} \tag{43}$$

for a horizontal tangent at point Q. It was suggested to take the average of these two dampings as the optimal value, which is given by

$$\zeta_{k\,\mathrm{opt}} = \sqrt{\frac{3\mu\phi_{mn}^2(x_k, y_k)}{8(1+\mu\phi_{mn}^2(x_k, y_k))}}. \tag{44}$$

NUMERICAL SIMULATION AND DISCUSSION

The optimum tuning and damping ratio of the vibration neutralizer for global control of a structure with closely spaced natural frequencies has been derived in the previous section. The main assumption is that the response of the structure in the vicinity of the natural frequency can be approximated by its corresponding mode. With such an assumption, the derivation was simply carried out by following the conventional fixed-points theory without any concern about its applicability. In this section, a series of simulations are presented and from these results, the applicability of the theory is then judged.

The effects of the optimal device on the kinetic energy of the structure in the vicinity of the natural frequency are simulated for three cases. These are when the natural frequencies are (a) relatively large-spaced; (b) relatively close-spaced, and (c) almost overlapping to each other. In the first case, although the natural frequencies are said to be “relatively large-spaced,” in principle they are closely spaced according to common understanding, for example in comparison with the investigation carried out by Dayou (2005, 2006) and Dayou and Wang (2004).

The structure being considered in this paper is a simply supported plate. This type of structure is an ideal approximation to many engineering applications and the control of its vibration is an important issue especially for the precise operation performances in aerospace systems, satellites, flexible manipulators, etc. (Benassi and Elliott, 2005a,b). In this investigation, a plate with the dimensions of $2\,\mathrm{m} \times 0.6\,\mathrm{m} \times 0.007\,\mathrm{m}$,

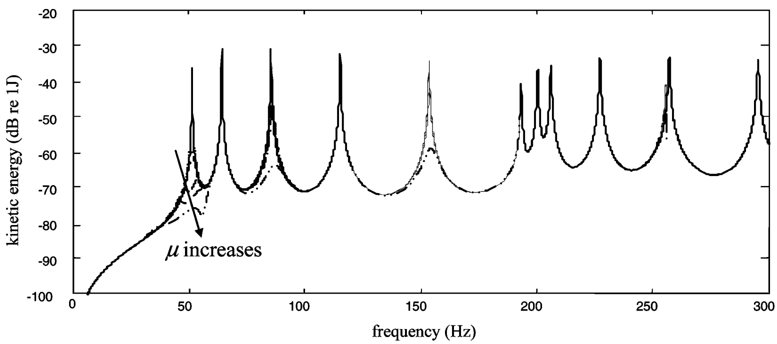
Table 1. Natural frequencies of the plate with corresponding m and n indexes and the location of the maximum deflection in x and y axis

Mode no.	Naturale frequency (Hz)	m, (maximum deflection)	n, (maximum deflection)
1	52	(1), $(0.5L_x)$	(1), $(0.5L_y)$
2	64	(2), $(0.25L_x, 0.75L_x)$	(1), $(0.5L_y)$
3	86	(3), $(L_x/6, 0.5L_x, 5L_x/6)$	(1), $(0.5L_y)$
4	115	(4), $(0.125L_x, 0.375L_x, 0.625L_x, 0.875L_x)$	(1), $(0.5L_y)$
5	154	(5), $(0.1L_x, 0.3L_x, 0.5L_x, 0.7L_x, 0.9L_x)$	(1), $(0.5L_y)$
6	193	(1), $(0.5L_x)$	(2), $(0.25L_y, 0.75L_y)$
7	201	(6), $(L_x/12, 3L_x/12, 5L_x/12, 7L_x/12, 9L_x/12, 11L_x/12)$	(1), $(0.5L_y)$
8	206	(2), $(0.25L_x, 0.75L_x)$	(2), $(0.25L_y, 0.75L_y)$
9	227	(3), $(L_x/6, 0.5L_x, 5L_x/6)$	(2), $(0.25L_y, 0.75L_y)$
10	256	(7), $(L_x/14, 3L_x/14, 5L_x/14, 7L_x/14, 9L_x/14, 11L_x/14, 13L_x/14)$	(1), $(0.5L_y)$
11	257	(4), $(0.125L_x, 0.375L_x, 0.625L_x, 0.875L_x)$	(2), $(0.25L_y, 0.75L_y)$
12	296	(5), $(0.1L_x, 0.3L_x, 0.5L_x, 0.7L_x, 0.9L_x)$	(2), $(0.25L_y, 0.75L_y)$

and with the following physical properties are used: Material density = 7870 kg/m^3, Young's modulus = 207E9 Pa, Poisson's ratio = 0.292, and modal damping = 0.001. The plate is subjected to unit amplitude of harmonic point primary force located at $(0.1L_x, 0.1L_y)$ where L_x and L_y are the length and the width of the plate, respectively. The natural frequencies of the plate are given in Table 1. Referring to this table, modes number 1, 2, and 3 were selected for the first case of investigation, modes 6, 7, and 8 for the second case, and modes 10 and 11 for the third case, respectively. The numerical simulation results for each case are described in the following section.

Relatively Large-Spaced Natural Frequencies

The first three modes, which are of concern in this investigation, are spaced between 12 Hz to 22 Hz and the fourth mode is 29 Hz above the third mode. Figures 1–3 show the effect of the optimal neutralizer on the kinetic energy of the plate in the vicinity of the first, second, and third natural frequency, respectively, when the mass ratio μ is changed.



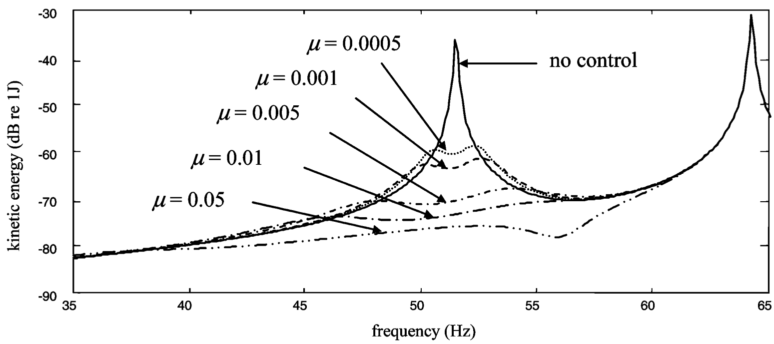


Figure 1. Effects of the optimal vibration neutralizer on the kinetic energy of the plate. The neutralizer is optimized to the first natural frequency and is applied at $(0.5L_x, 0.5L_y)$. (a) Overall kinetic energy and (b) kinetic energy in the vicinity of the first natural frequency.

The neutralizer is placed at the location with the highest deflection amplitude, as close to the center of the plate as possible. Otherwise, the highest deflection point closest to the primary force is selected. In each figure, the first graph (a) shows the kinetic energy of the plate up to 300 Hz, whereas the second graph (b) shows the kinetic energy in the vicinity of each natural frequency being considered. The first graph shows the overall effect of the optimal neutralizer whereas the second graph is shown for clear visualization of each natural frequency considered.

Generally, the kinetic energy for each targeted natural frequency is relatively smooth when the optimal vibration neutralizer is applied. It can be seen from each figure that no new resonance occurs in the whole frequency range and this is the desirable result when using the fixed-points theory. As the neutralizer's mass increases, the kinetic energy decreases to a level where the effect of the dominant mode is removed leaving only the effect from the residual modes. In Fig. 1(b), for example, with the first natural frequency as the control target,

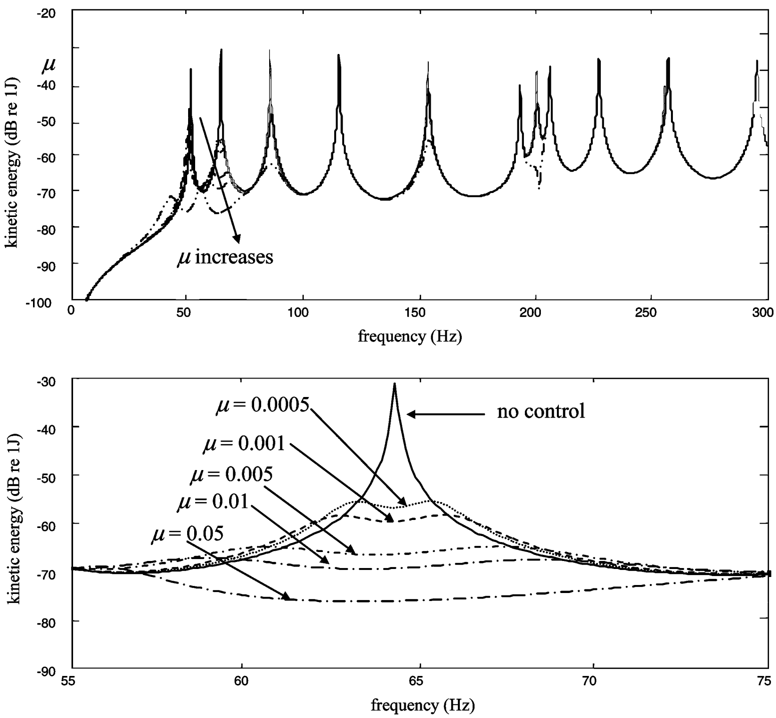


Figure 2. Effects of the optimal vibration neutralizer on the kinetic energy of the plate where the neutralizer is optimized to the second natural frequency and is applied at $(0.25L_x, 0.5L_y)$. (a) Overall kinetic energy and (b) kinetic energy in the vicinity of the second natural frequency.

the vibrational effect from the first mode is almost removed when $\mu = 0.01$. Similar observation is shown in Figs. 2(b) and 3(b) when the control target is the second and the third natural frequency, respectively.

Besides having a relatively smooth kinetic energy in the vicinity of the targeted natural frequency, the structure also has a relatively flat kinetic energy with the application of the optimal vibration neutralizer, especially for the second and third mode shown in Figs. 2 and 3, respectively. However, the kinetic energy curves for the first mode in Fig. 1(a) is skewed where the kinetic energy is higher at the higher frequency compared to the lower frequency. This is because of the strong influence from the higher mode, which is the second mode. This phenomenon is not observed for the second and third modes (Figs. 2 and 3) because they are equally affected by their respective lower and higher neighboring modes.

It is worthwhile to mention here that after a certain value of the mass ratio, the kinetic energy curves are no longer smooth. This can be

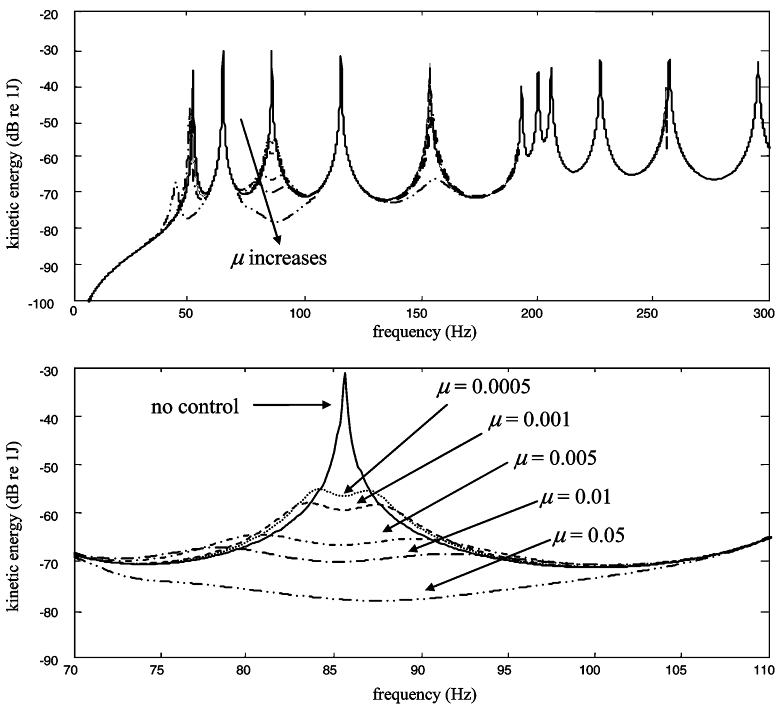


Figure 3. Kinetic energy of the plate with neutralizer optimized to its third natural frequency and is applied at $(0.5L_x, 0.5L_y)$. (a) Overall kinetic energy and (b) kinetic energy in the vicinity of the third natural frequency.

clearly seen in all figures when $\mu = 0.05$. Although no new resonance appears, the result becomes difficult to predict and therefore, in real application, the neutralizer's parameters must be properly selected prior to its fabrication.

The application of the vibration neutralizer optimized to a specific mode also has some effects on other modes. For example, for the neutralizer that was optimized to the first natural frequency shown in Fig. 1(a), some reductions in kinetic energy at the higher modes, which are the third and fifth mode, are also observed. For the neutralizer that was optimized to the second natural frequency [Fig. 2(a)], some reductions in the kinetic energy can be observed on the first, third, fifth, and seventh modes, whereas for the neutralizer optimized to the third natural frequency [Fig. 3(a)], reductions can also be observed on the first and fifth modes. The reduction of the modes other then the targeted mode increases as the neutralizer's mass increases. However, there are modes where no reductions can be observed in the kinetic energy. For example, for the graph shown in Fig. 1(a), no reduction is observed on the second, fourth, sixth mode, and so on. This is because the neutralizer

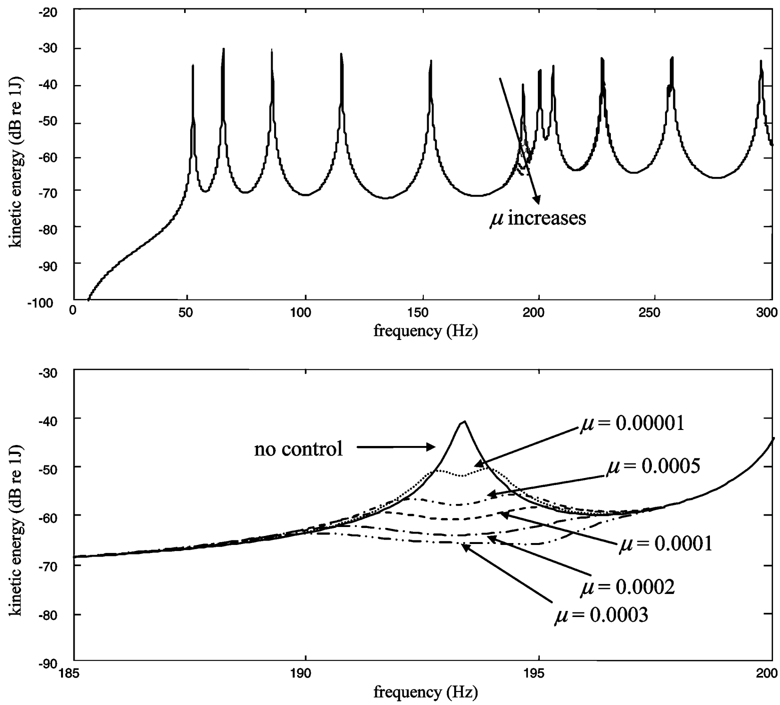


Figure 4. Effects of the neutralizer that was optimized to the sixth natural frequency on the kinetic energy of the plate. The neutralizer is applied at $(0.5L_x, 0.25L_y)$. (a) Overall kinetic energy and (b) kinetic energy in the vicinity of the sixth natural frequency.

is attached at the location which coincides with the nodal point of the modes concerned. The same reason applies to Figs. 2(a) and 3(a).

The effect of the neutralizer on the kinetic energy other than at the targeted mode would be interesting to investigate. However, this is outside the scope of this paper and therefore is not discussed in detail. The discussion is focused on the applicability of the theory for global control of a structure with closely spaced natural frequencies.

Relatively Close-Spaced Natural Frequencies

In the previous section, it has been proven that the fixed-points theory can be used to determine the optimum tuning and damping ratios of the neutralizer that may remove the effects of the dominant mode in the kinetic energy of relatively large-spaced natural frequencies of the plate. In this section, the use of the theory for relatively close-spaced natural frequencies is discussed in comparison with the previous application.

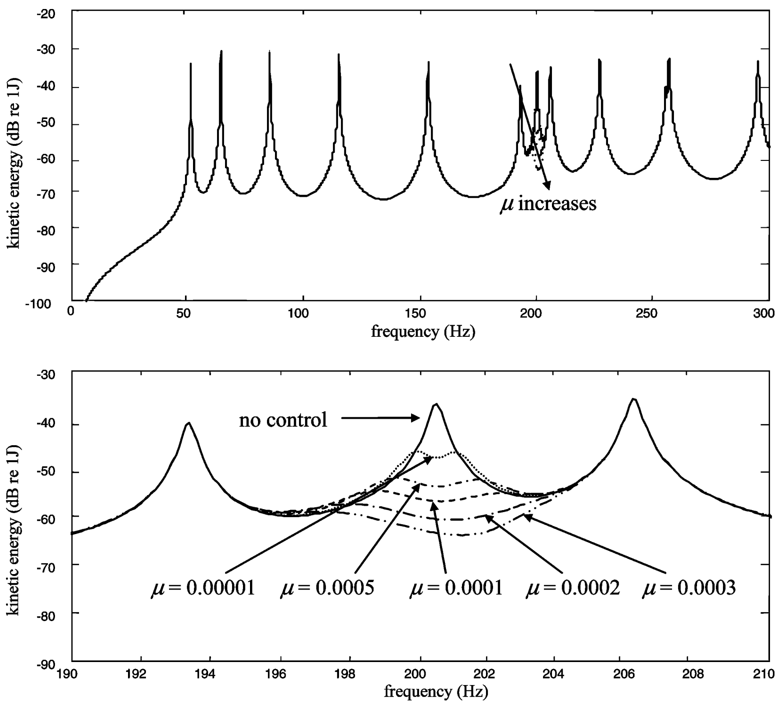


Figure 5. Effects of the neutralizer that was optimized to the seventh natural frequency on the kinetic energy of the plate. The neutralizer is applied at $(5L_x/12, 0.5L_y)$. (a) Overall kinetic energy and (b) kinetic energy in the vicinity of the seventh natural frequency.

For this purpose, modes number six, seven, and eight were chosen where the frequency spacing is 8 Hz (between sixth and seventh) and 5 Hz (for seventh and eighth). Clearly, the selected natural frequencies are very close to each other.

The kinetic energy of the plate with neutralizer optimized to the sixth, seventh, and eighth mode is shown in Figs. 4, 5, and 6, respectively. Compared to the previous case, all of the kinetic energy curves for each mode are skewed towards their neighboring mode that has higher kinetic energy. For example, for the sixth mode, the kinetic energy curves are skewed toward the seventh mode. This is because the kinetic energy of the seventh mode is higher than the sixth mode. Similar explanation applies to modes seven and eight.

Although all of the kinetic energy curves are skewed, they are still smooth and this shows that the theory may also be used for the case considered. There is no new resonance in the whole frequency range of interest which is the desirable result when using this theory. As the neutralizer's mass increases, the reduction in the kinetic energy also

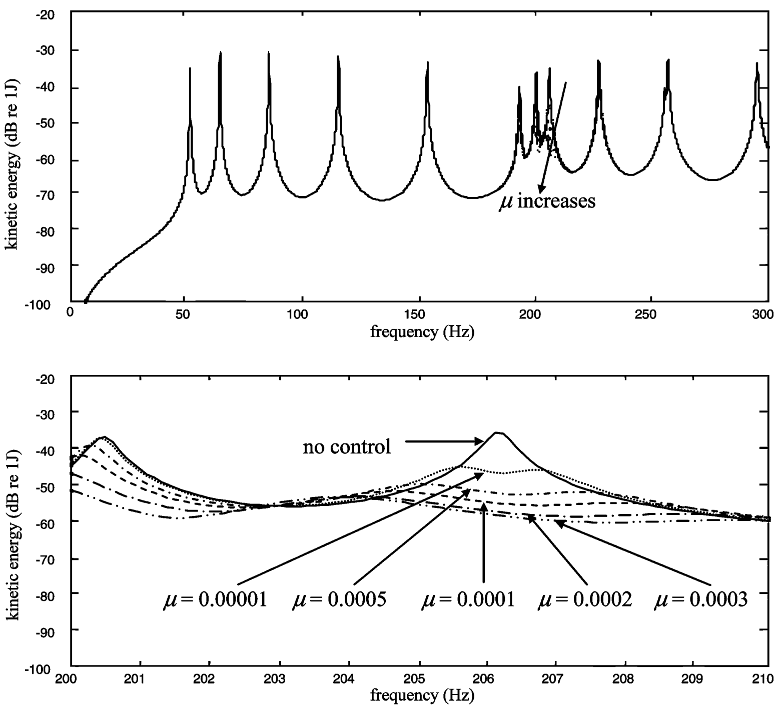


Figure 6. Effect of the neutralizer that was optimized to the eight, natural frequency on the kinetic energy of the plate. The neutralizer is applied at $(0.25L_x, 0.25L_y)$. (a) Overall kinetic energy and (b) kinetic energy in the vicinity of the eighth natural frequency.

increases as shown in Figs. 4–6. However, after a certain value of neutralizer mass, the kinetic energy curve becomes less smooth where the curve is lower when it is close to the targeted natural frequency. Nevertheless, this does not influence the usefulness of the theory, as the kinetic energy is greatly reduced in the vicinity of the targeted natural frequency depending on the neutralizer's mass.

Almost Overlapping Natural Frequencies

For this case, modes number 10 and 11 were chosen where their natural frequencies are spaced by only 1 Hz. The close ups of the kinetic energy at and around the natural frequencies are shown in Figs. 7 and 8. In Fig. 7, when the neutralizer is optimized to the 10th mode and is placed at $(0.25L_x, 0.25L_y)$ and at a smaller mass ratio, higher reduction in the kinetic energy can be observed at the natural frequency of the targeted mode compared to its both sides. As the mass is increased, this

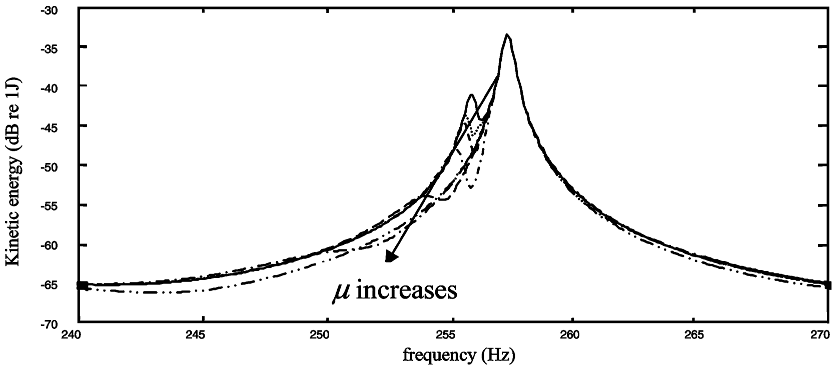


Figure 7. Effects of the neutralizer that was optimized to the tenth natural frequency on the kinetic energy of the plate. The neutralizer is applied at $(0.25L_x, 0.25L_y)$ and $\mu = 0.005, 0.0005, 0.00005, 0.000005, 0.0000005,$ and 0.0000001.

effect disappears where reduction can only be observed along the curve where the 10th mode dominates (i.e., along the left hand side curve). Similar effect is seen in Fig. 8 where, as the neutralizer's mass increases, reduction can only be seen along the curve where the 11th mode dominates (i.e., along the right hand side curve). However, in Fig. 8, at higher neutralizer mass, the kinetic energy at the natural frequency of the 10th mode increases compared to the kinetic energy without the control device.

Although the reduction in the kinetic energy is not as high as in previous cases, the neutralizer still has the ability to remove the effect

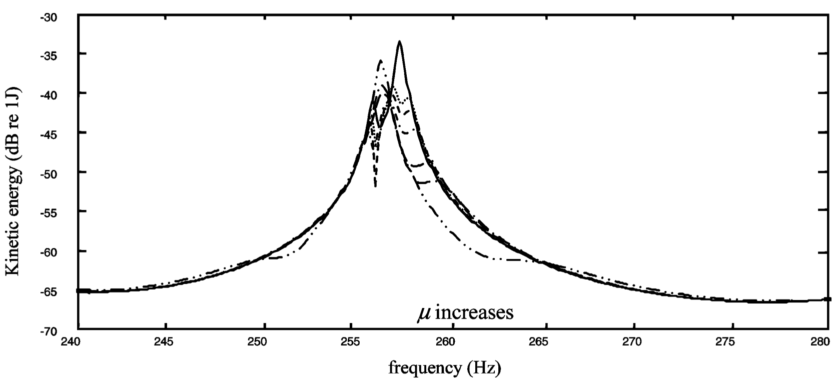


Figure 8. Effects of the neutralizer that was optimized to the eleventh natural frequency on the kinetic energy of the plate. The neutralizer is applied at $(0.375L_x, 0.25L_y)$ and $\mu = 0.0005, 0.00005, 0.00003, 0.00001, 0.000005,$ and 0.000003.

of the targeted mode. However, the reduction can only be observed in a narrow range of frequency where a single mode dominates. In most frequency ranges, the two modes contributed equally to the kinetic energy of the plate, and therefore it could be quite difficult to make a single neutralizer very effective at both modes at the same time.

SUMMARY AND CONCLUSION

In this paper, the use of the fixed-points theory was developed and examined on a structure which has closely-spaced natural frequencies. The effectiveness of the vibration neutralizer was investigated by means of some numerical simulations. In general, the theory can be used to determine the optimum tuning and damping ratios of the vibration neutralizer that remove the effects from the dominant mode leaving only the effects from residual modes in the global behavior of the structure. However, the optimal neutralizer becomes less effective as the spacing between the natural frequencies gets small.

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