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Analytical and Experimental Investigation of Partially Covered Piezoelectric Cantilever Energy Harvester

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Electrical energy is normally generated through different sources such as hydroelectric, wind, heat, nuclear transformation, chemical reactions or vibrations. Nowadays, harvesting power from mechanical vibration is one of the novel technologies that usually can be done by systems based on electromagnetic, electrostatic, piezoelectric and combination of them. Piezoelectric systems can convert motion from the vibrating structures into electrical power. Cellulose Electro-active paper (EAPap) has been recognized as a novel smart piezoelectric material that can be used for energy harvesting purposes. One of the most prevalent method for vibration energy harvesting is using unimorph piezoelectric cantilever beams. In this paper, an analytical solution based on distributed parameter model is presented to calculate the generated energy from vibration of cantilever substrate that is partially covered by EAPap material. In the studied structure, piezoelectric layer thickness in comparison to the length of the beam and thickness of substrate material can be considered very thin. Thus its effect on the vibration behavior of structure is negligible. The results are validated by experimental values. The analytical data was found to be very close to experimental results and finite element simulation values. Findings from this study provide guidelines on system parameters that can be manipulated for more efficient performance in different ambient source conditions.

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1. Introduction

Energy harvesting is deriving energy from external sources, capturing, and storing for small, wireless autonomous devices. It is also known as power harvesting or power scavenging. Energy harvesters usually generate a very small amount of power for low-power electronic devices such as wireless sensor networks (WSNs) and wearable electronics. One of the main advantages of the energy harvesters is that the energy source of power scavengers is free and present in the ambient surroundings, while the input fuel costs to other common methods in large-scale is relatively high. Power harvesting from ambient vibrations, light, wind or heat can lead to indefinite life of the smart sensors. For example, there is a large amount of mechanical vibrations in the environment coming from a plethora of sources, among them road and rail traffic, vibrating components of machinery, industrial plants, construction works, and aircrafts.

Several academic and commercial groups have focused on design, analysis and development of vibration energy harvesting technology. The Vibration power scavenger is designed to incessantly power wireless sensor nodes used in industrial and constructional monitoring and measurement applications. Using suitable electronic devices, Vibration power scavengers can be used for creating self-powered systems, such as self-powered sensors for monitoring of highway bridges. This is of particular interest whenever the use of batteries is not desired (because of the high costs of maintenance) and a power supply via cable is not possible (such as rotorcraft rotor and other dynamic parts of machines).[1-3]

Piezoelectric materials can convert kinetic energy into electrical energy. These materials can generate electric charge in response to an applied deformation. There are many effective parameters in amount of generated power from piezoelectric materials, such as type and amount of loading, frequency of vibrations and etc. So far, more than 200 appropriate materials have been found and are made that have different applications. Hence, much attention is required in the choice of piezoelectric material. Cellulose Electro-active paper (EAPap) is a novel smart material which its efficiency in energy harvesting applications has proven, recently.[4,5]

Cellulose EAPap is made by coating thin electrodes on both sides of cellulose paper. Induced charge is produced when the paper is deformed. The main advantage of cellulose over other piezoelectric polymers that are obtained from a chemical process, is its abundance. This material can extracted from prevalent natural resources such as plants, seaweed and cotton. Very recently, EAPap has been discovered and has started to receive much attention due to its huge potential for various piezoelectric energy harvesters as well as sensors and actuators. The advantages of the cellulose EAPap material over other piezoelectric materials include low price, large displacement output, low actuation voltage, dryness, low power consumption, flexibility, sensing capability and biodegradable characteristics.[4,6-8]

In most prevalent energy harvester designs, piezoelectric material can be used as an additional layer that may cover some part of the beams to harvest vibration energy for autonomous self-sufficient power sensors. In similar circumstances, among the beams, a cantilever structure has the maximum deflection and thus the maximum stress, strain and consequently a higher power output. Therefore, the cantilever beam structure is used in most vibration energy harvester designs.[9,10] For the first time, Erturk and Inman developed the distributed parameter model and presented the analytical solution to the coupled problem of a piezoelectric power scavenger based on the Euler-Bernoulli beam assumptions. In previous research works, the coupled vibration response of the harvester explicitly for harmonic base excitations in the form of translation with small rotation and the coupled voltage response across the resistive load were obtained for fully covered piezoelectric unimorph and bimorph configurations.[9,11-13]

This article presents a piecewise solution of a piezoelectric cantilever energy harvester and investigates the output voltage, current and power of a cantilever beam that is partially covered by EAPap piezoelectric material. Due to the desired coverage of beam by piezoelectric material, this study offers a more comprehensive solution than previous works based on the distributed parameter model. The analytically obtained electromechanical expressions are used in an experimental parametric case study that is composed of an aluminum cantilever partially covered by EAPap piezoelectric material.

2. Free Vibration Analysis

In this section, the estimations of natural frequencies for a rectangular cantilever beam that is partially covered by a Nanoscale-Layered piezoelectric material, is presented. The schematic of piezoelectric energy scavenging system is pictured in Fig. 1. As can be seen, h_p is the piezoelectric layer thickness, h_s is the substrate layer thickness, L is the length of the cantilever beam and x_1 and x_2 are distance of beginning and end of the piezoelectric layer from the base. It is noteworthy that due to the small thickness of piezoelectric layer compared to those of the substrate layer (usually less than 2% of the substrate layer), the effect of piezoelectric electro-active paper on the vibration characteristic of the structure is negligible and can be ignored. Therefore regardless of piezoelectric layer effects, the structure vibration behavior can be considered such as a simple cantilever beam.

For a simple unimorph cantilever beam that undergoes undamped free vibration, the first natural frequency that plays a major role in the

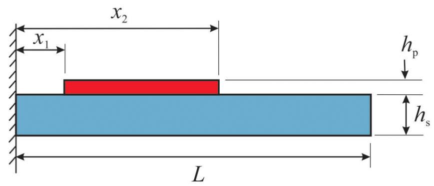

Fig. 1 Schematic drawing of energy scavenging system

vibration behavior of structure, can be obtained by using Rayleigh method as follows;[14,15]

$$\omega_1 = \frac{0.5678}{L^2}\sqrt{\frac{\frac{E_s h_s^3}{12}+\frac{2E_p h_p^3}{3}+\frac{E_p h_p h_s^2}{2}+E_p h_s h_p^2}{\rho_s h_s + 2\rho_p h_p}} \quad (1)$$

The Eq. (1) is a rule of thumb for calculating the main resonance frequency of cantilever energy harvesters. In this equation, E_s and E_p are young's modulus and ρ_s and ρ_p are material density for substrate and piezoelectric layers, respectively. For a simple cantilever beam regardless of the thickness of the piezoelectric ($h_p \approx 0$), the relation can be expressed as;

$$\omega_1 = 0.1639\frac{h_s}{L^2}\sqrt{\frac{E_s}{\rho_s}} \quad (2)$$

It should be noted that the first mode of mechanical vibration has the lowest resonant frequency, and typically provides the most deflection and therefore output power. Accordingly, vibration power scavengers are generally designed to operate in the first resonant mode of frequency.[16]

There are also more complex relationships for expressing the resonance frequency of the systems based on the Rayleigh-Ritz method at k-th mode as follows;[17]

$$\omega_k = \lambda_k^2\sqrt{\frac{EI}{mL^4}} \quad (3)$$

In this relation, λ_k is eigenvalue in k-th mode, and m and EI are mass per length and flexural rigidity of the beam, respectively. m can be written as below;

$$m = B\rho_s h_s \quad (4)$$

The term EI for a simple cantilever beam is expressed as;

$$EI = \frac{EBh_s^3}{12} \quad (5)$$

The frequency equation for finding λ_k is as follows;

$$1+\cos\lambda_k\cosh\lambda_k = 0 \quad (6)$$

It is necessary to mention that mass normalized eigenfunction of a clamped-free beam representing the k-th mode shape corresponding to the undamped free vibration problem is found out to be;[9]

$$w_k(x) = \left(\cos\left(\frac{\lambda_k}{L}x\right)-\cosh\left(\frac{\lambda_k}{L}x\right)\right) - \frac{\cos(\lambda_k)+\cosh(\lambda_k)}{\sin(\lambda_k)+\sinh(\lambda_k)}\left(\sin\left(\frac{\lambda_k}{L}x\right)-\sinh\left(\frac{\lambda_k}{L}x\right)\right) \quad (7)$$

3. Forced Vibration Analysis

The analytical solution for the cantilever energy harvester that is partially covered by piezoelectric material is based on Euler-Bernoulli or thin beam theory. The general equation of motion for a cantilever under the influence of base excitation can be expressed as;[2,18,19]

$$\frac{\partial^2 M(x,t)}{\partial x^2}+m\frac{\partial^2 z_{rel}(x,t)}{\partial t^2}=-m\frac{\partial^2 z_b(x,t)}{\partial t^2} \tag{8}$$

where $M(x, t)$ is the internal moment, m is the mass per unit length of the beam, $z_b(x, t)$ is the base motion of the beam and $z_{rel}(x, t)$ is the transverse displacement of the neutral axis (at point x and time t) relative to its base due to bending.

The stress-strain relations for both substrate and piezoelectric layers can be expressed as;

$$\sigma_1^p = E_p(\varepsilon_1^p - d_{31}E_3) \tag{9}$$

$$\sigma_1^s = E_s\varepsilon_1^s \tag{10}$$

where σ is the stress, ε is the strain, d_{31} is the piezoelectric strain constant, and E_3 is the applied electrical field. E_3 can be written in terms of voltage $v(t)$ as below;

$$E_3(t) = -v(t)/h_p \tag{11}$$

The bending strain, ε at a certain level, y from the neutral axis of the beam is simply proportional to the curvature of the beam at position x and can be expressed as;

$$\varepsilon_1 = -y\frac{\partial^2 z_{rel}(x,t)}{\partial x^2} \tag{12}$$

The width of the piezoelectric layer is assumed to be the same as the width of the substrate layer, denoted by B. The internal moment $M(x, t)$, can be written as;[12]

$$M(x,t) = -\int_{-(h_s/2)}^{(h_s/2)}\sigma_1^s By\,dy - \int_{(h_s/2)}^{(h_s/2)+h_p}\sigma_1^p By\,dy \tag{13}$$

Employing Eqs. (9), (10), and (12) into Eq. (13), one may obtain;

$$M(x,t) = EI\frac{\partial^2 z_{rel}(x,t)}{\partial x^2}+\varpi v(t) \tag{14}$$

where;

$$\varpi = -E_p B d_{31}(h_s+h_p) \tag{15}$$

Using Heaviside step function, the internal moment can be written as;

$$M(x,t) = EI\frac{\partial^2 z_{rel}(x,t)}{\partial x^{22}}+\varpi v(t)[H(x-x_1)-H(x-x_2)] \tag{16}$$

Heaviside step functions are added in Eq. (16) to ensure the survival of this term when the internal moment expression, $M(x, t)$ is used in the differential equation of motion. The Heaviside function is the integral of the Dirac delta function $d(x)$. Employing Eq. (16) into Eq. (8) yields;

$$\begin{aligned}&EI\frac{\partial^4 z_{rel}(x,t)}{\partial x^4}+m\frac{\partial^2 z_{rel}(x,t)}{\partial t^2}+\varpi v(t)\left[\frac{d\delta(x-x_1)}{dx}-\frac{d\delta(x-x_2)}{dx}\right]\\&=-m\frac{\partial^2 z_b(x,t)}{\partial t^2}\end{aligned} \tag{17}$$

On the other hand, to obtain electrical circuit equation with mechanical coupling, the following piezoelectric constitutive relation is used;

$$D_3 = d_{31}\sigma_1+\varepsilon_{33}^T E_3 \tag{18}$$

where D_3 is the electrical displacement and e_{33}^T is the permittivity at constant stress. The permittivity at constant strain e_{33}^S, replaces the permittivity component through $e_{33}^T = e_{33}^S + d_{31}E_p$. Thus Eq. (18) can be rewritten as;

$$D_3 = d_{31}E_p\varepsilon_1(x,t)-\varepsilon_{33}^S\frac{v(t)}{h_p} \tag{19}$$

The average bending strain can be expressed as;

$$\varepsilon_1(x,t) = -\left(\frac{h_s}{2}+h_p\right)\frac{\partial^2 z_{rel}(x,t)}{\partial x^2} \tag{20}$$

The accumulated electric charge in the cellulose EAPap film, $q(t)$, is obtained by integrating the electrical displacement over the piezoelectric area from x_1 to x_2;

$$\begin{aligned}q(t) &= \int \overrightarrow{D_3}\cdot\tilde{n}\,dA\\&=-\int_{x=x_1}^{x=x_2}\left(d_{31}E_p\left(\frac{h_s}{2}+h_p\right)B\frac{\partial^2 z_{rel}(x,t)}{\partial x^2}+\varepsilon_{33}^S B\frac{v(t)}{h_p}\right)dx\end{aligned} \tag{21}$$

where $\tilde{n}$ is unit outward normal and $\vec{D}$ is the vector of electric displacement. Clearly, the nonzero terms of these vectors are the ones in 3 direction.

Then, the current generated by the cellulose EAPap, $i(t)$, can be given by;

$$\begin{aligned}i(t) &= \frac{dq(t)}{dt}\\&-\int_{x=x_1}^{x=x_2}\left(d_{31}\left(\frac{h_s}{2}+h_p\right)E_p B\frac{\partial^3 z_{rel}(x,t)}{\partial x^2\partial t}\right)dx-\frac{\varepsilon_{33}^S B(x_2-x_1)}{h_p}\frac{dv(t)}{dt}\end{aligned} \tag{22}$$

As the terms imply, the first component of the current function is due to the vibratory motion of the cantilever and the second component includes the voltage across the cellulose EAPap film. The term (e_{33}^S B $(x_2 - x_1) / h_p$) is called the capacitance of the EAPap film and is connected to the resistive load (R_L). Therefore, the voltage output can be obtained as;

$$\begin{aligned}v(t) &= R_L i(t)\\&=-R_L\left[\begin{aligned}&\int_{x=x_1}^{x=x_2}\left(d_{31}\left(\frac{h_s}{2}+h_p\right)E_p B\frac{\partial^3 z_{rel}(x,t)}{\partial x^2\partial t}\right)dx\\&+\frac{\varepsilon_{33}^S B(x_2-x_1)}{h_p}\frac{dv(t)}{dt}\end{aligned}\right]\end{aligned} \tag{23}$$

Rearranging the Eq. (23) leads to electrical circuit equation as below;

$$\frac{dv(t)}{dt}+\frac{h_p}{\varepsilon_{33}^S B(x_2-x_1)R_L}v(t) = -\frac{d_{31}\left(\frac{h_s}{2}+h_p\right)E_p h_p}{\varepsilon_{33}^S(x_2-x_1)}\int_{x=x_1}^{x=x_2}\frac{\partial^3 z_{rel}(x,t)}{\partial x^2\partial t}dx \tag{24}$$

Eq. (24) is the general distributed parameter electromechanical equation for a cantilevered piezoelectric power scavenger in transverse vibrations that can be used for energy harvesters that partially or fully covered by piezoelectric films.

The relative vibratory motion of the cantilever ($z_{rel}(x, t)$) can be represented by an absolutely and uniformly convergent series of the eigenfunctions as follows;

$$z_{rel}(x,t)=\sum_{k=1}^{\infty}w_k(x)q_k(t) \tag{25}$$

Using free vibration solution, the integral term in Eq. (24) can be written as;

$$\int_{x=x_1}^{x=x_2}\frac{\partial^3 z_{rel}(x,t)}{\partial x^2\partial t}dx=\sum_{k=1}^{\infty}\frac{dq_k(t)}{dt}\int_{x_1}^{x_2}\frac{d^2w_k(x)}{dx}dx = \sum_{k=1}^{\infty}\frac{dq_k(t)}{dt}\left[\frac{dw_k(x)}{dx}\bigg|_{x=x_2}-\frac{dw_k(x)}{dx}\bigg|_{x=x_1}\right] \tag{26}$$

Accordingly, Eq. (24) can be rewritten as below;

$$\frac{dv(t)}{dt}+\frac{h_p}{\varepsilon_{33}^S B(x_2-x_1)R_L}v(t)=\sum_{k=1}^{\infty}\psi_k\frac{dp_k(t)}{dt} \tag{27}$$

where;

$$\psi_k(x)=-\frac{d_{31}E_p h_p(h_s/2+h_p)}{\varepsilon_{33}^s(x_2-x_1)}\left[\frac{dw_k(x)}{dx}\bigg|_{x=x_2}-\frac{dw_k(x)}{dx}\bigg|_{x=x_1}\right] \tag{28}$$

Eq. (27) is a differential equation that can be solved by the following integrating factor;

$$\eta(t)=e^{t/\tau_c} \tag{29}$$

where τ_c is the circuit time constant and can be expressed by;

$$\tau_c=\frac{R_L\varepsilon_{33}^s B(x_2-x_1)}{h_p} \tag{30}$$

Combining Eq. (17) and Eq. (25) leads to;

$$EI\frac{\partial^4}{\partial x^4}\left[\sum_{k=1}^{\infty}w_k(x)q_k(t)\right]+m\frac{\partial^2}{\partial t^2}\left[\sum_{k=1}^{\infty}w_k(x)q_k(t)\right] + \varpi v(t)\left[\frac{d\delta(x-x_1)}{dx}-\frac{d\delta(x-x_2)}{dx}\right]=-m\frac{\partial^2 z_b(x,t)}{\partial t^2} \tag{31}$$

Integrating Eq. (31) over the length of the beam after multiplying it with both sides by mass-normalized eigenfunction, $w_p(x)$, using orthogonality condition and finally adding a proportional damping to actualize equation,[20] gives the equation of motion in modal space as follows;

$$\frac{d^2q_k(t)}{dt^2}+2\zeta_k\omega_k\frac{dq_k(t)}{dt}+\omega_k^2q_k(t)+\upsilon_k v(t) = -\int_0^L w_k(x)m\frac{\partial^2 z_b(x,t)}{\partial t^2}dx \tag{32}$$

where ϑ_k represents the modal coupling term and can be written as;

$$\upsilon_k=\varpi\left[\frac{dw_k(x)}{dx}\bigg|_{x=x_2}-\frac{dw_k(x)}{dx}\bigg|_{x=x_1}\right] \tag{33}$$

Modal damping ratio can be estimated from the frequency response function curve using half power bandwidth method for each natural frequency. In this method, FRF amplitude of the system is obtained first. Corresponding to each natural frequency, there is a peak in FRF amplitude. 3 dB down from the peak there are two point corresponding to half power point. The more the damping, the more the frequency range between the points. Half-power bandwidth BD is defined as the ratio of the frequency range between the two half power points to the natural frequency at this mode. For the first natural frequency, the common method is logarithmic decrement.[21] For harmonic oscillation of cantilever, base motion and output voltage can be written as $z_b = Y_0e^{j\omega t}$ and $v(t) = V_0e^{j\omega t}$ respectively. Thus $q_k(t)$ becomes;

$$q_k(t)=\frac{\left[m\omega^2Y_0\int_0^L w_k(x)dx-\upsilon_k V_0\right]e^{j\omega t}}{\omega_k^2-\omega^2+2j\zeta_k\omega_k\omega} \tag{34}$$

where j is the imaginary number sign and ω is the driving frequency. Also substituting $v(t) = V_0e^{j\omega t}$ in Eq. (27) leads to;

$$\left(\frac{1+j\omega\tau_c}{\tau_c}\right)V_0e^{j\omega t}=\sum_{k=1}^{\infty}\psi_k\frac{dq_k(t)}{dt} \tag{35}$$

Differentiating $q_k(t)$ from Eq. (34) and substituting in Eq. (35) gives the voltage amplitude across the resistance;

$$\left(\frac{1+j\omega\tau_c}{\tau_c}\right)V_0e^{j\omega t} = \sum_{k=1}^{\infty}\psi_k\frac{j\omega\left[m\omega^2Y_0\int_0^L w_k(x)dx-\upsilon_k V_0\right]e^{j\omega t}}{\omega_k^2-\omega^2+2j\zeta_k\omega_k\omega} \tag{36}$$

Thus, the ratio of the voltage output to the base acceleration or better known as the voltage FRF, is given by;

$$\frac{v(t)}{\omega^2Y_0e^{j\omega t}}=\frac{\sum_{k=1}^{\infty}\frac{jm\omega\psi_k\left(\int_0^L w_k(x)dx\right)}{\omega_k^2-\omega^2+2j\zeta_k\omega_k\omega}}{\left(\sum_{k=1}^{\infty}\frac{j\omega\upsilon_k\psi_k}{\omega_k^2-\omega^2+2j\zeta_k\omega_k\omega}\right)+\frac{1+j\omega\tau_c}{\tau_c}} \tag{37}$$

If the beam is excited around the natural frequency of the k-th mode, the main contributions in the summation signs appearing in Eqs. (36) and (37) are from the k-th mode. Often piezoelectric cantilever energy harvesters are designed to operate mainly at their first resonant frequency (i.e., $k = 1$). Accordingly, when the natural frequencies of the system are well separated, it can be useful to simplify the equation for the fundamental vibration mode of the cantilever. The reduced expression for the voltage across the resistive load can be written as;

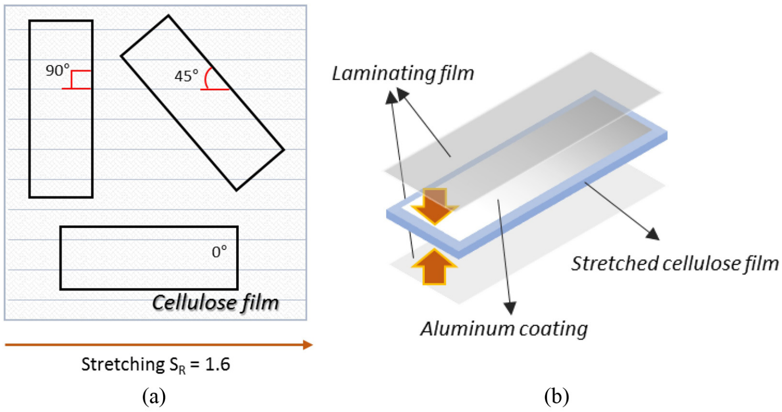

Fig. 2 Schematic diagram showing the (a) cutting orientation of the EAPap film and (b) its coating procedure

$$v^1(t) = \frac{jm\tau_c\omega^3\psi_1\left(\int_0^L w_1(x)dx\right)}{j\omega\upsilon_1\psi_1\tau_c + 1 + j\omega\tau_c\omega_1^2 - \omega^2 + 2j\zeta_1\omega_1\omega} Y_0 e^{j\omega t} \tag{38}$$

where the superscript 1 denotes that the respective expression is reduced for excitation around the fundamental vibration mode. The Eq. (38) can be rewritten as;

$$v^1(t) = \left|V_0^1\right| e^{j(\omega t + \theta_v)} \tag{39}$$

where the amplitude of the voltage output in the first natural frequency can be written as follows;

$$\left|V_0^1\right| = \frac{Y_0 m\tau_c\omega^3\left|\psi_1\left(\int_0^L w_1(x)dx\right)\right|}{\sqrt{\begin{matrix}[\omega_1^2 - \omega^2 1 + 2\tau_c\zeta_1\omega_1]^2 \\ +[2\zeta_1\omega_1\omega + \tau_c\omega\upsilon_1\psi_1 + \omega_1^2 - \omega^2]^2\end{matrix}}} \tag{40}$$

and the phase angle between the reduced voltage output and the base displacement can be obtained as;

$$\begin{aligned}\theta_v = {} & \frac{\pi}{2}sgn\left(\psi_1\left(\int_0^L w_1(x)dx\right)\right) \\ & -\tan^{-1}\left(\frac{2\zeta_1\omega_1\omega + \tau_c\omega\upsilon_1\psi_1 + \omega_1^2 - \omega^2}{\omega_1^2 - \omega^2 1 + 2\tau_c\zeta_1\omega_1}\right)\end{aligned} \tag{41}$$

where "*sgn*" is the *sign function*.

The reduced expression for the current flow through the resistive load can be written as;

$$i^1(t) = \left|I_0^1\right| e^{j(\omega t + \theta_i)} \tag{42}$$

where the amplitude of the current output in the first natural frequency is as below;

$$\left|I_0^1\right| = \frac{Y_0 m\tau_c\omega^3\left|\psi_1\left(\int_0^L w_1(x)dx\right)\right|}{R_L\sqrt{\begin{matrix}[\omega_1^2 - \omega^2 1 + 2\tau_c\zeta_1\omega_1]^2 \\ +[2\zeta_1\omega_1\omega + \tau_c\omega\upsilon_1\psi_1 + \omega_1^2 - \omega^2]^2\end{matrix}}} \tag{43}$$

and the phase angle between the reduced current output and the base displacement is same as phase angle between the reduced voltage output and the base displacement ($\theta_i = \theta_v$). Similarly, the amplitude of output power can be expressed as;

$$\left|P_0^1\right| = \frac{\left(Y_0 m\tau_c\omega^3\left|\psi_1\left(\int_0^L w_1(x)dx\right)\right|\right)^2}{R_L\left(\begin{matrix}[\omega_1^2 - \omega^2 1 + 2\tau_c\zeta_1\omega_1]^2 \\ +[2\zeta_1\omega_1\omega + \tau_c\omega\upsilon_1\psi_1 + \omega_1^2 - \omega^2]^2\end{matrix}\right)} \tag{44}$$

It is necessary to mention that Eqs. (38) to (44) are valid only at the excitation frequencies around the fundamental natural frequency of the cellulose EAPap-based vibration energy harvester. Replacing the subscripts and superscripts 1 by k can lead to respective electrical and mechanical expressions around the k-th natural frequency.

4. Experimental Analysis

4.1 Preparation of EAPap Piezoelectric film

Cellulose is a smart material that can be used for many applications, such as micro flying objects, micro insect robots, MEMS, biosensors, and flexible electrical displays.[22] This material exhibits piezoelectric effect and it is shown concluded that the piezoelectricity of cellulose-based EAPap can be used in energy transduction applications.[4] Also EAPap film has a good reversibility for mechanical performance, such as bending strain and displacement, under electric field. The fabrication process of thin cellulose EAPap film was well explained in previous articles.[23,24] So far, it is fabricated by using separated processes: Cellulose solution fabrication, tape casting, washing with deionized (DI) water/ isopropyl alcohol (IPA) mixture, stretching and drying.[7] Herein the process of fabrication in this experiment is explained briefly for clarity.

EAPap piezoelectric film is regenerated cellulose that is fabricated from raw cotton with degree of polymerization of 4500. The cotton that was tore into smallest pieces possible, dissolved in anhydrous N, N-dimethyl acetamide (DMCAc) (Sigma Aldrich) together with lithium chloride (LiCl) with specific ratio at 110ºC. To produce homogeneous solution, the mixture was stirred using magnetic bar stirrer until LiCl and the raw cotton palp completely dissolved. It was then centrifuged to produce transparent and highly viscous solution. After that, it was poured on a clean glass plate and casted uniformly using doctor blade. Deionized water and isopropyl alcohol (IPA) solutions were used to

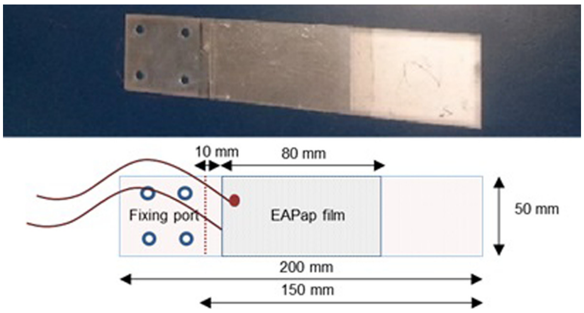

Fig. 3 Photograph and schematic diagram of the EAPap piezoelectric energy harvester

remove Li+(DMAc)x microcations residuals. While it is wet, the cellulose was stretched at 1.6 ratios from its original length which was subsequently dried using infrared light for approximately one hour. At this point, a thin regenerated cellulose film of 1.5×10^{-5} m thickness is produced with piezoelectric property, which is known as EAPap piezoelectric film. For experimental use, the film was cut at 45 degree relative to the stretching direction (see Fig. 2(a)) with the required dimensions. For measurement purpose, aluminum electrodes were deposited on the both sides of the film using thermal evaporating method. To prevent short circuit between the electrodes and the host as well as to reduce the possibility of damage, each EAPap film was laminated with transparent films (refer to Fig. 2(b)). The EAPap sample was fabricated to have the length 8 cm and width of 5 cm.

4.2 Preparation of EAPap Piezoelectric energy harvester

The EAPap piezoelectric energy harvester was fabricated in the form of a cantilever beam. Aluminum beam with the length of 200 mm, width of 50 mm and thickness of 1 mm was used as the host structure for capturing the ambient vibration energy by bending of the structure. The beam's length is inclusive of 5 cm fixing part that has four holes for screw fastening, and the EAPap film with the dimensions of 80 mm length and 50 mm width attached 10 mm away from the fixing line (refer to Fig. 3). The EAPap film was attached near to its clamped base where the largest bending was found.

4.3 Measurement procedure

The EAPap piezoelectric energy harvester in the form of cantilever beam (or simply EAPap piezobeam) was fixed on the bobbin of an electromagnetic shaker (Eliezer HEV-50) with tightening jig. The piezobeam was exited with 100 mV input voltage controlled using a function generator (Agilent 33220A) and amplifier (Eliezer EA157) in the frequency range of interest. This is corresponding to 2 mm of displacement input. An accelerometer was used to monitor the displacement input where the input voltage was adjusted whenever necessaryto maintain the displacement. The experimental arrangement is shown in Fig. 4.

A potentiometer was used as resistive external load, added in series to the EAPap film for the measurement of power output. A picoammeter (Keithley 6485) was used to measure the electric current in the circuit, and a pulse analyzer (Bruel & Kjaer 35360B-030) was used to monitor voltage output. The impedance of EAPap film was measured using LCR meter (HP 4282A) and with respect to the frequency changes.

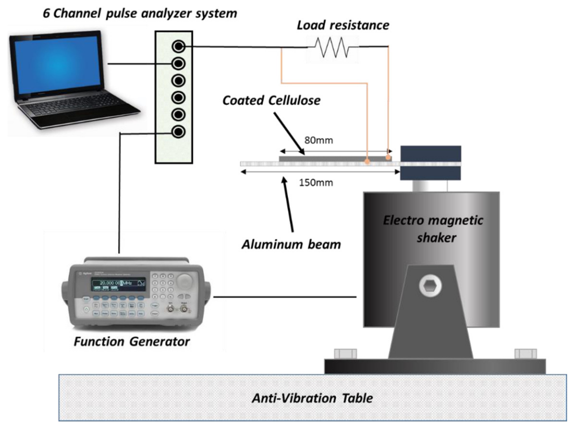

Fig. 4 Experimental setup for the measurement of voltage and current and subsequently the harvested power from the excited EAPap piezoelectric energy harvester

Table 1 First natural frequency of the cellulose EAPap-based energy harvester based on different analytical and experimental methods

	Rayleigh method based on Eq. (2)	Rayleigh-Ritz method based on Eq. (3)	Experiment (Hz)
Natural frequency (Hz)	34.6	36.3	35.8
Relative error (%)	3.4	1.4	-

5. Results, Validation and Discussion

5.1 Resonance frequency of cellulose EAPap-based power scavenger

Most energy harvesting devices developed to date are based on first resonance frequency of the system and it is determined that the built-in voltage could lead to significant change in output power over one order of magnitude higher when the vibration frequency and the mechanical resonance frequency of the system are matched together. Therefore, estimating resonant frequency of the system is a key parameter in designing process of mechanical energy harvesters.

The resonance frequency of the system is calculated based on the theoretical equations developed in previous research works by submitting the properties of substrate (Young's Modulus, E = 69 GPa, density of the beam, ρ = 2700 kgm^{-3}, thickness of the beam, h_s = 0.1 cm and length of the beam, L = 15 cm) in Eqs. (2) and (3).[14,15] Afterward the results are compared with the empirical resonance frequency in Table 1. As mentioned in,[15] the fundamental natural frequency of a rectangular cantilever beam is constant for different width but same length, thickness and materials, because Eqs. (2) and (3) is independent of the width of the cantilever beam. As it can be seen in Table 1, there is a good agreement between the results and the error is almost 3%. The error might due to the experimental errors and some external factors such as the way of fastening and fixing the cantilever to the shaker.

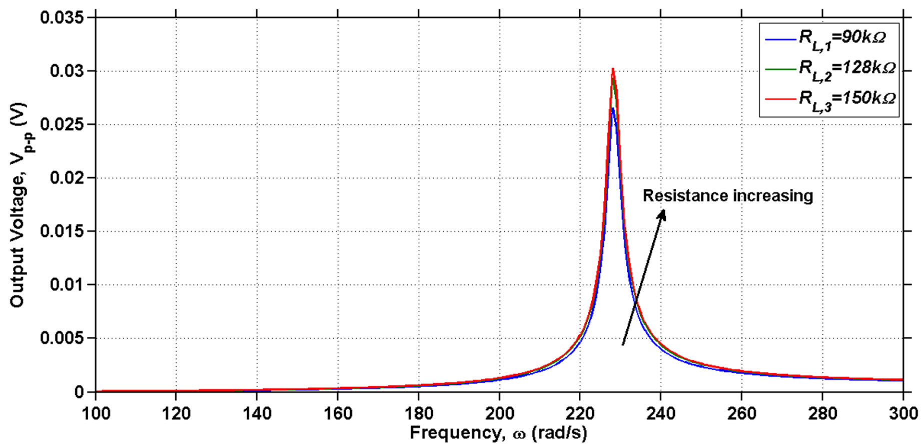

Fig. 5 Peak-to-peak voltage of the EAPap-based energy harvester for a range of frequencies with different external resistive loads

5.2 Output of EAPap energy harvester

In any power scavenging circuit, impedance matching of the external load to the energy harvester is necessary for reaching the maximum output power. In the experiment, a simple circuit was constructed to include a potentiometer used as a variable external load so that it could be varied to match the source impedance (i.e., impedance of the cellulose EAPap films). Prototype has been characterized under steady-state sinusoidal excitation of the shaker and is subjected to its resonance frequency, so the output electric current was sinusoidal.

The EAPap cantilever beam with length of 15 cm, width of 5 cm and with impedance-matching external load of 90 kΩ was connected to the harvesting circuit. The peak-to-peak current, I_{p-p}, was measured by the picoammeter as 284 nA at resonance frequency of 35.8 Hz. Measuring the root mean square (rms) current, I_{rms}, and the resistive load, R_{load}, it is possible to calculate the mean power output, P_{mean}, using the following equation;

$$P_{mean} = I_{rms}^2 \times R_{load} = 0.354 \times I_{p-p}^2 \times R_{load} \quad (45)$$

The peak-to-peak voltage, V_{p-p}, and peak-to-peak current, I_{p-p}, was measured to be 25.6 mV and 284 nA, respectively. It is notable that $V_{p-p} = 2V_0$ and $I_{p-p} = 2I_0$. Also the maximum mean power output, P_{mean}, for the harvester was found to be 0.907 nW as the resistance of the external load was 90 kΩ and damping ratio of the structure was 0.006876. The Table 2 shows the theoretical values and the experimental measurements. As can be seen, the results are in a good agreement, yielding little relative error. Accordingly the theoretical results are validated and can be generalized for a range of frequencies, resistive loads and damping ratios.

The theoretical output voltage, output current and mean power output was derived for damping ratio of 0.006876 and three different external loads $R_{L,1} = 90000$, $R_{L,2} = 128000$, $R_{L,3} = 150000$ for a range of frequencies between 100 rad/s to 300 rad/s (about 15.92 Hz to 47.75 Hz) and the results can be seen in Figs. 5, 6 and 7, respectively. As can be seen, the harvested power is significant around the resonance frequency and the value of load resistance R_L is an important parameter that shapes the dynamic behavior of the system. The system is expected to move toward the short circuit conditions for low values of load

Table 3 Analytical and experimental results of the V_{p-p}, I_{p-p} and P_{mean} for partially covered cellulose-based piezoelectric energy harvester

	V_{p-p} (mV)	I_{p-p} (nA)	P_{mean} (nW)
Experimental results	25.6	284	0.9071
Theoretical values	26.6	295	0.9831
Relative error (%)	3.9	3.9	8.4

resistance ($R_L \to 0$), whereas the open circuit behavior is expected for large values of load resistance ($R_L \to \infty$). For every excitation frequency, increasing the load resistance can lead to increasing the output voltage and the maximum voltage is achieved in open circuit condition. Also increasing the load resistance can lead to decreasing the output current. The output power behavior against resistive load is more complicated than the output voltage and the output current. Because the output power is product of the output voltage and the output current, but the voltage and the current have opposite behavior by changing the resistive load. Although the behavior of power with changing load resistance is not monotonic,[12] but plotting the graph can lead to optimal choice of resistance in different case studies. For example in this case study, choosing 90 kΩ resistance, leads to higher output power and more harvested energy. More discussion about resistive load effects can be found in Refs. 25 and 26.

Also the peak-to-peak voltage, V_{p-p}, peak-to-peak current, I_{p-p}, and mean power output, P_{mean}, of the energy harvester was investigated with a range of damping ratios, $\xi_1 = 0.004$, $\xi_2 = 0.006876$ and $\xi_3 = 0.009$ by the developed theoretical equations. Although the structural damping is unchangeable for this structure, but it gives a good insight for the future researches and designs. It is notable that the width reduction of the cantilever, will not change the vibration behavior of the system, but can change the damping ratio. Accordingly, contrary to the expectations, the damping ratio somewhat can be controlled by changing the width of the cantilever.[14,15,27,28] As can be seen in Figs. 8, 9 and 10, all of the electrical parameters (V_{p-p}, I_{p-p} and P_{mean}) have similar behavior against damping ratio. The figures show that decreasing the damping ratio is a desired option in designing the piezoelectric energy harvesters and leads to increasing output voltage, current and the power simultaneously.

According to figures drawn and discussion, all designs that are

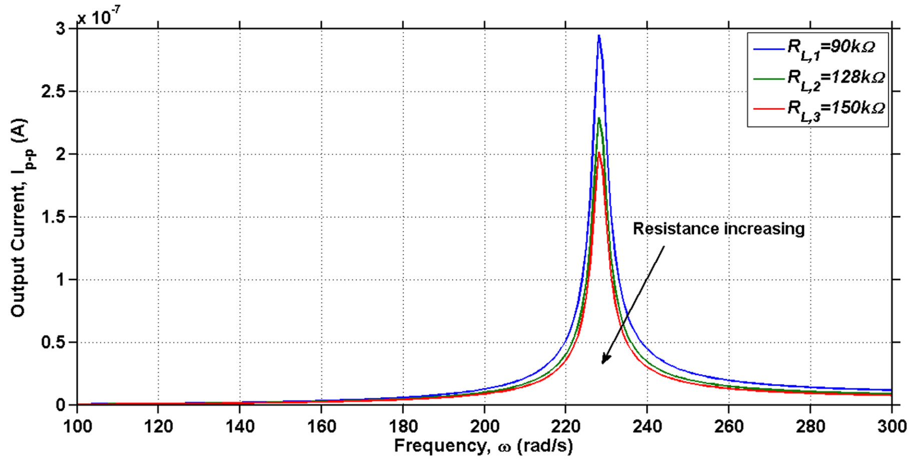

Fig. 6 Peak-to-peak output current of the EAPap-based energy harvester for a range of frequencies in different external resistive loads

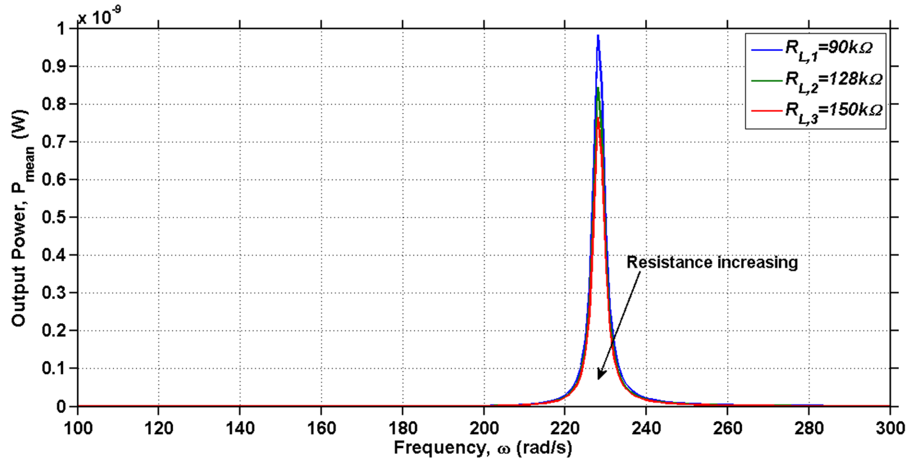

Fig. 7 Mean power output of the EAPap-based energy harvester for a range of frequencies with different external resistive loads

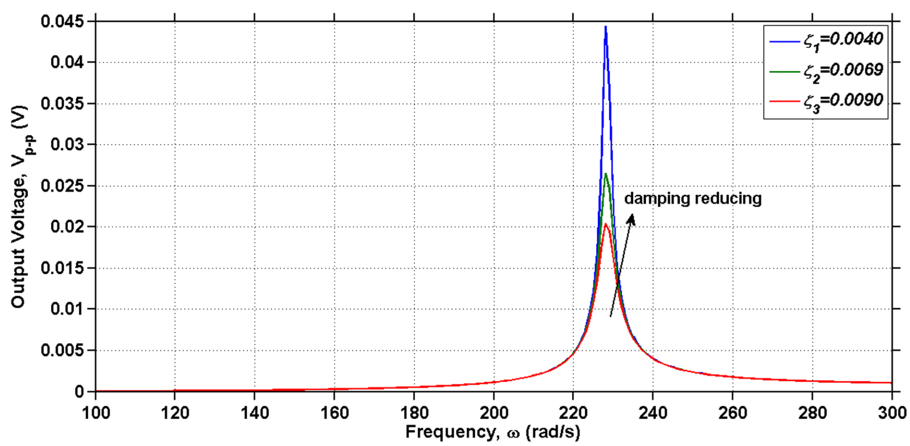

Fig. 8 Peak-to-peak output voltage of the EAPap-based energy harvester for a range of frequencies in different damping ratios

leading to lower damping, can increase the efficiency of the energy harvester. For example, width reduction decreases the damping ratio and keeps the vibration behavior of the system, constant.[27] So it can be concluded that the smaller beams with lower damping ratios, vibrate at higher amplitudes and they can increase the harvested energy from the piezoelectric power scavengers.

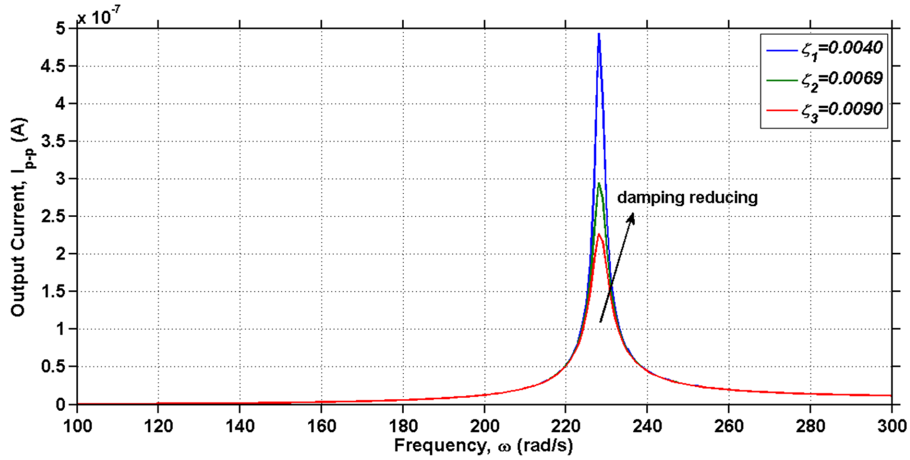

Fig. 9 Peak-to-peak output current of the EAPap-based energy harvester for a range of frequencies in different damping ratios

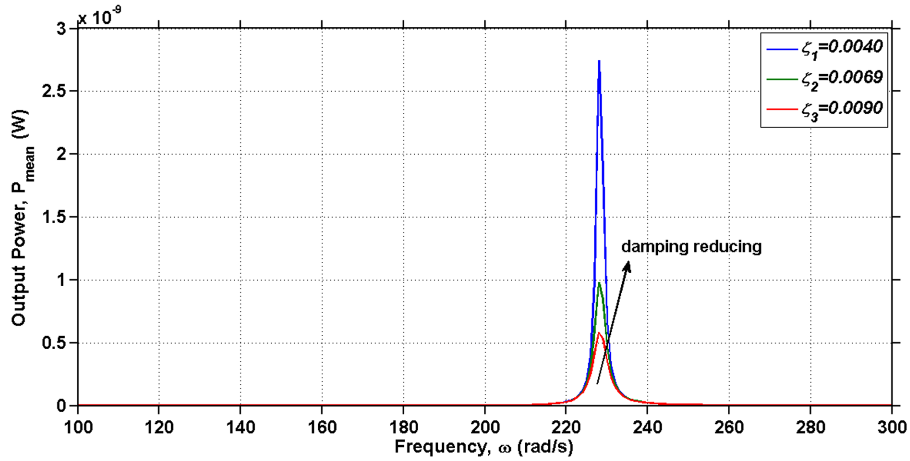

Fig. 10 Mean power output of the EAPap-based energy harvester for a range of frequencies in different damping ratios

6. Summary and Conclusion

Typically, a cantilevered harvester beam that is partially or fully covered by a piezoelectric film layer is located on a vibrating host structure and the harvester beam generates electrical power due to base excitation. Recently, the authors have presented the closed-form analytical solution for a cantilever energy harvesters that are fully covered by piezoelectric layers based on the Euler-Bernoulli beam assumptions. In this work, the analytical solution is extended to configurations that are partially covered by piezoelectric films and experimentally validated. In mechanical modeling, the damping effect is considered. The developed equations are further reduced for the case of excitation around a resonance frequency. The analytically obtained expressions are then used in a parametric case study with a novel piezoelectric material that has started to receive much attention due to its huge potential for various piezoelectric energy harvesters and is called EAPap. The values are validated by experimental results and then output voltage, current and the power output are plotted against frequency for different external resistive loads and damping ratios in the first natural frequency. Although the behavior of power with changing load resistance is not monotonic, all of the considered electrical outputs behave monotonically and similarly to change of the damping ratio. The results show that decreasing the damping ratio is a desired option in designing the piezoelectric energy harvesters and leads to increasing output voltage, current and the power simultaneously. Width reduction is an applied method that can lead to maintaining the fundamental natural frequency of the beam at a constant value and increase the output harvested power. Width-split method is a practical way for increasing the electromechanical coupling and therefore the electrical outputs of the harvester.

Future works will consider design optimization of the cellulose EAPap-based energy harvesters by changing the location of the piezoelectric layer and size of load resistance. Also width-split method will be investigated to achieve the optimal geometry.

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