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Parameter identification of partially covered piezoelectric cantilever power scavenger based on the coupled distributed parameter solution

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ABSTRACT

Among the various techniques of power scavenging, piezoelectric energy harvesting usually has more power density. Although piezo-ceramics are usually more efficient than other piezoelectric materials, since they are very brittle and fragile, researchers are looking for alternative materials. Recently Cellulose Electro-active paper (EAPap) has been recognized as a smart material with piezoelectric behavior that can be used in energy scavenging systems. The majority of researches in energy harvesting area, use unimorph piezoelectric cantilever beams. This paper presents an analytical solution based on distributed parameter model for partially covered pieoelectric cantilever energy harvester. The purpose of the paper is to describe the changes in generated power with damping and the load resistance using analytical calculations. The analytical data are verified using experiment on a vibrating cantilever substrate that is partially covered by EAPap films. The results are very close to each other. Also asymptotic trends of the voltage, current and power outputs are investigated and expressions are obtained for the extreme conditions of the load resistance. These new findings provide guidelines for identification and manipulation of effective parameters in order to achieve the efficient performance in different ambient source conditions.

1. Introduction

There are small amounts of wasted energy that could be useful if captured. Recovering even a fraction of this energy would have a significant economic and environmental impact. This is where energy harvesting (EH) comes in. Energy harvesting, or energy scavenging, is a process that captures small amounts of energy that would otherwise be lost as heat, light, sound, vibration or movement. The captured energy can be used for improving the efficiency and enabling modern technologies such as wireless and batteryless sensor networks. This is because EH can be an alternative for batteries in

low power electronic devices. Power scavenging from ambient vibrations, light, heat or wind could enable smart sensors to be functional indefinitely. For example, there is a large amount of mechanical vibrations in the environment coming from a plethora of sources, among them industrial plants, road and rail traffic, construction works, and aircrafts. Vibration energy harvesting is an attractive technique for potential powering of low-energy electronics and wireless sensors.

Many recent studies have focused on analysis and development of mechanical power scavengers. The study of efficiency of vibration energy harvesters has become an important aspect of the researches. Even long-lasting batteries have a limited lifespan and must be replaced every few years. The batteries replacement is expensive when there are hundreds of wireless sensors in remote locations. The purpose of vibration energy harvester design is to permanently power industrial wireless sensor nodes (WSN) used in different monitoring and measurement applications. Using efficient designs for increasing the power density can move us closer towards batteryless wireless sensors networks and systems. This is of particular interest whenever supplying power via cable is not possible and the use of batteries and the associated maintenance expenditure are not desired [1–3].

Kinetic energy can be converted into electrical energy by means of the piezoelectric effect. Piezo elements convert the kinetic energy from vibrations or shocks into electrical energy. Piezoelectric power scavengers generate electricity depending on the amount of force used in compressing or deforming the material, the amount and type of deformation of the material's crystal structure and the speed or frequency of compressions or vibrations to the material. There are more than 200 appropriate materials which need careful selection for the particular application. Cellulose Electro-active paper (EAPap) has been recognized as a new smart material that can be used for energy harvesting purposes.

EAPap is made with cellulose paper by coating thin electrodes on both sides of it. This paper can produce induced charge in the presence of a bending or longitudinal strain. Compared to other piezoelectric polymers obtained from a chemical process, cellulose is an abundant natural material from common natural resources such as plants, cotton and seaweed. Very recently, EAPap has been discovered and has started to receive much attention due to its huge potential for various piezoelectric energy harvesters as well as sensors and actuators. This EAPap material has many advantages in terms of large displacement output, low actuation voltage, low power consumption, dryness, low price, flexibleness, sensing capability and biodegradable characteristics [4–7].

One such method is to use piezoelectric material as an additional layer that may cover some part of the beams to harvest vibration energy for self-powered sensors. Compared to other structural forms of beams, a cantilever beam can obtain the maximum deformation under the same conditions. The larger deflection leads to more stress, strain, and consequently a higher output harvested power. Therefore, the vast majority of piezoelectric vibration energy harvesting devices uses a cantilever beam structure. Erturk and Inman developed the distributed parameter model and presented the analytical solution to the coupled problem of a piezoelectric energy harvester configuration based on the Euler–Bernoulli beam assumptions. In previous research works, the coupled vibration response of the harvester explicitly for harmonic base excitations in the form of translation with small rotation and the coupled voltage

response across the resistive load were obtained for fully covered piezoelectric unimorph and bimorph configurations [8–11].

The analytical method for partially covered piezoelectric energy harvester is discussed before in [12] and the results are validated using experimental analysis. The aim of this paper is developing the solution for partially covered piezoelectric energy harvesters and discussing the load resistance size and damping effects on the output voltage, current and power of a cantilever beam that is partially covered by EAPap piezoelectric material in a wide range of frequencies. It can be observed that there is an optimum value for load resistance and also reducing the damping can increase the harvested energy throughout the frequency domain. The results of the analytically obtained electromechanical expressions are validated using an experimental study on a partially covered piezoelectric cantilever energy harvester.

2. Fundamentals of the coupled distributed parameter model

The proposed piezoelectric energy scavenging system is schematically shown in Figure 1. In this picture, h_p is the piezoelectric layer thickness, h_s is the substrate layer thickness, L is the length of the cantilever beam and x_1 and x_2 are distance of beginning and end of the piezoelectric layer from the base. For a thin piezoelectric layer compared to the substrate layer, the effect of piezoelectric layer on the vibration characteristic of the structure is negligible and can be ignored. Therefore the vibration behavior of the structure can be considered such as a simple cantilever beam by the substrate alone.

The main resonance frequency of a simple unimorph cantilever that undergoes undamped free vibration, can be written as [13,14];

$$\omega_1 = \frac{0.5678}{L^2} \sqrt{\frac{\frac{E_s h_s^3}{12} + \frac{2E_p h_p^3}{3} + \frac{E_p h_p h_s^2}{2} + E_p h_s h_p^2}{\rho_s h_s + 2\rho_p h_p}} \tag{1}$$

ρ_s and ρ_p are material density and E_s and E_p are young's modulus for substrate and piezoelectric layers, respectively. If the thickness of the piezoelectric film is much smaller compared to the substrate

($h_p \approx 0$), this relationship can be expressed as;
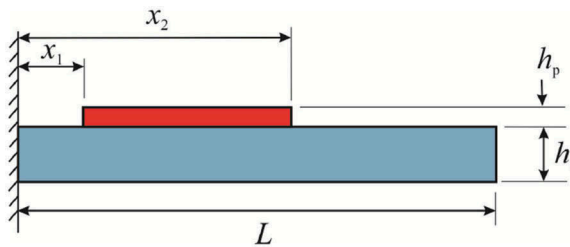
Figure 1. Cantilever power scavenging system that is partially covered by Nanoscale-Layered piezoelectric material.

$$\omega_1 = 0.1639 \frac{h_s}{L^2} \sqrt{\frac{E_s}{\rho_s}} \tag{2}$$

It should be noted that the first mode of mechanical vibration has the lowest resonance frequency, and typically provides the most deflection and therefore, the output power. Accordingly, vibration power scavengers are generally designed to operate in the first resonant mode of frequency.

The Euler-Bernoulli method is used to model the cantilever beam. The general equation of motion for a cantilever under the base excitation and zero initial conditions can be written as [2,15,16];

$$\frac{\partial^2 M(x,t)}{\partial x^2} + m \frac{\partial^2 z_{rel}(x,t)}{\partial t^2} = -m \frac{\partial^2 z_b(x,t)}{\partial t^2} \tag{3}$$

where $z_b(x,t)$ is the base motion of the beam and $z_{rel}(x,t)$ is the transverse displacement of the neutral axis relative to its base due to bending. Also, $M(x,t)$ and m are the internal moment and the mass per unit length of the beam, respectively.

From the materials mechanics, the equation between strain, ε at a certain level, y and beam deflection can be obtained as:

$$\varepsilon_1 = -y \frac{\partial^2 z_{rel}(x,t)}{\partial x^2} \tag{4}$$

For same width of piezoelectric and substrate layers (denoted by B), the internal bending moment term in Equation (3) is the first moment of axial strain over the cross-section:

$$M(x,t) = -\int_{-(h_s/2)}^{(h_s/2)} \sigma_1^s B y\, dy - \int_{(h_s/2)}^{(h_s/2)+h_p} \sigma_1^p B y\, dy \tag{5}$$

where σ is the stress. Employing stress-strain relations for both substrate and piezo-electric layers, and substituting Equation (4). into Equation (5), one may obtain;

$$M(x,t) = EI \frac{\partial^2 z_{rel}(x,t)}{\partial x^2} + \phi v(t) \tag{6}$$

where

$$\phi = -E_p B d_{31}(h_s + h_p) \tag{7}$$

with d_{31} is the piezoelectric strain constant and $v(t)$ is the voltage.

For the survival of the second term of Equation (6). when the internal moment expression, $M(x,t)$ is used in the differential equation of motion, heaviside step function must be added to that term. Adding this term and employing into the Equation (3). yields;

$$EI \frac{\partial^4 z_{rel}(x,t)}{\partial x^4} + m \frac{\partial^2 z_{rel}(x,t)}{\partial t^2} + \phi v(t) \left[\frac{d\delta(x-x_1)}{dx} - \frac{d\delta(x-x_2)}{dx} \right] = -m \frac{\partial^2 z_b(x,t)}{\partial t^2} \tag{8}$$

Unknowns of the equations, are determined using the second constitutive equation of piezoelectric material that is,

$$D_3 = d_{31}E_p\varepsilon_1(x,t) - \varepsilon_{33}^S \frac{v(t)}{h_p} \tag{9}$$

where D_3 is the electrical displacement and ε_{33}^S is the permittivity at constant strain. The average bending strain can be expressed as;

$$\varepsilon_1(x,t) = -\left(\frac{h_s}{2} + h_p\right)\frac{\partial^2 z_{rel}(x,t)}{\partial x^2} \tag{10}$$

The charge collected on the electrode surface, $q(t)$, can be expressed as the electrical displacement integral on the area of the surface from x_1 to x_2 according to:

$$q(t) = \int \vec{D}_3.\tilde{n}dA = -\int_{x=x_1}^{x=x_2}\left(d_{31}E_p(\frac{h_s}{2} + h_p)B\frac{\partial^2 z_{rel}(x,t)}{\partial x^2} + \varepsilon_{33}^S B\frac{v(t)}{h_p}\right)dx \tag{11}$$

where $\vec{D}$ is the vector of electric displacement and $\tilde{n}$ is unit outward normal. The voltage of the piezoelectric layer is calculated after multiplying electric resistance R_L by generated current, which is the first derivative of electric charge with respect to time.

$$v(t) = R_L i(t) = -R_L\left[\int_{x=x_1}^{x=x_2}\left(d_{31}(\frac{h_s}{2} + h_p)E_pB\frac{\partial^3 z_{rel}(x,t)}{\partial x^2\partial t}\right)dx + \frac{\varepsilon_{33}^S B(x_2-x_1)}{h_p}\frac{dv(t)}{dt}\right] \tag{12}$$

As the terms imply, the first component of the voltage function in the bracket is due to the vibratory motion of the cantilever and the second component includes the voltage across the cellulose EAPap film. The term $(\varepsilon_{33}^S B(x_2 - x_1)/h_p)$ is called the capacitance of the EAPap film and is connected to the resistive load R_L.

Rearranging the Equation (12). leads to;

$$\frac{dv(t)}{dt} + \frac{h_p}{\varepsilon_{33}^S B(x_2-x_1)R_L}v(t) = -\frac{d_{31}(\frac{h_s}{2} + h_p)E_ph_p}{\varepsilon_{33}^S(x_2-x_1)}\int_{x=x_1}^{x=x_2}\frac{\partial^3 z_{rel}(x,t)}{\partial x^2\partial t}dx \tag{13}$$

Equations (11) and (13) are two coupled governing differential equations of a variable width piezoelectric beam,which can be solved by implementing eigenfunction expansion method as

Equation (13) is a governing equation of a piezoelectric beam energy scavenger in transverse vibrations and can be solved by implementing eigenfunction expansion method. So the relative vibratory motion of a beam ($z_{rel}(x,t)$) can be expressed as:

$$z_{rel}(x,t) = \sum_{k=1}^{\infty} w_k(x)q_k(t) \tag{14}$$

Using free vibration solution, the integral term in Equation (13). can be written as;

$$\int_{x=x_1}^{x=x_2}\frac{\partial^3 z_{rel}(x,t)}{\partial x^2\partial t}dx = \sum_{k=1}^{\infty}\frac{dq_k(t)}{dt}\int_{x_1}^{x_2}\frac{d^2w_k(x)}{dx}dx$$

$$= \sum_{k=1}^{\infty}\frac{dq_k(t)}{dt}\left[\frac{dw_k(x)}{dx}\bigg|_{x=x_2} - \frac{dw_k(x)}{dx}\bigg|_{x=x_1}\right] \tag{15}$$

Accordingly, Equation (13). can be rewritten as below;

$$\frac{dv(t)}{dt} + \frac{h_p}{\varepsilon_{33}^S B(x_2 - x_1)R_L}v(t) = \sum_{k=1}^{\infty} \varpi_k \frac{dq_k(t)}{dt} \tag{16}$$

where;

$$\varpi_k(x) = -\frac{d_{31}E_p h_p(h_s/2 + h_p)}{\varepsilon_{33}^S(x_2 - x_1)}\left[\frac{dw_k(x)}{dx}\bigg|_{x=x_2} - \frac{dw_k(x)}{dx}\bigg|_{x=x_1}\right] \tag{17}$$

Equation (16) can be solved for $v(t)$ by using the following integrating factor;

$$\lambda(t) = e^{t/\tau_c} \tag{18}$$

where τ_c is the time constant of the circuit and can be written as below;

$$\tau_c = \frac{R_L \varepsilon_{33}^S B(x_2 - x_1)}{h_p} \tag{19}$$

Combining Equation (8). and Equation (14). result in;

$$EI\frac{\partial^4}{\partial x^4}\left[\sum_{k=1}^{\infty} w_k(x)q_k(t)\right] + m\frac{\partial^2}{\partial t^2}\left[\sum_{k=1}^{\infty} w_k(x)q_k(t)\right]$$
$$+ \phi v(t)\left[\frac{d\delta(x - x_1)}{dx} - \frac{d\delta(x - x_2)}{dx}\right]$$
$$= -m\frac{\partial^2 z_b(x,t)}{\partial t^2} \tag{20}$$

Using the Galerkin method, coupled partial differential equations of system can be transformed to ordinary differential equations. For this purpose, both sides of the Equation (20). must be multiplied with mass-normalized eigenfunction, $w_p(x)$, and integrating over the length of the beam, using orthogonality condition and finally adding a proportional damping to actualize equation [17,18], gives the equation of motion in modal space as below;

$$\frac{d^2 q_k(t)}{dt^2} + 2\zeta_k\omega_k\frac{dq_k(t)}{dt} + \omega_k^2 q_k(t) + \gamma_k v(t) = -\int_0^L w_k(x)m\frac{\partial^2 z_b(x,t)}{\partial t^2}dx \tag{21}$$

where γ_k represents the modal coupling term;

$$\gamma_k = \phi\left[\frac{dw_k(x)}{dx}\bigg|_{x=x_2} - \frac{dw_k(x)}{dx}\bigg|_{x=x_1}\right] \tag{22}$$

For calculating modal damping ratio in the first natural frequency, the logarithmic decrement method is usually employed [19]. For harmonic oscillation of beam, output voltage and base motion can be written as $v(t) = V_0 e^{j\omega t}$ and $z_b = Y_0 e^{j\omega t}$ respectively. So $q_k(t)$ is;

$$q_k(t) = \frac{\left[m\omega^2 Y_0 \int_{x_1}^{x_2} w_k(x)dx - \gamma_k V_0\right]e^{j\omega t}}{\omega_k^2 - \omega^2 + 2j\zeta_k\omega_k\omega} \tag{23}$$

where ω is the driving frequency and j is the imaginary number sign. Also substituting $v(t) = V_0 e^{j\omega t}$ in Equation (16). leads to;

$$\left(\frac{1+j\omega\tau_c}{\tau_c}\right)V_0 e^{j\omega t} = \sum_{k=1}^{\infty}\varpi_k \frac{dq_k(t)}{dt} \tag{24}$$

Combining Equation (23). and Equation (24). gives the voltage amplitude across the resistance;

$$\left(\frac{1+j\omega\tau_c}{\tau_c}\right)V_0 e^{j\omega t} = \sum_{k=1}^{\infty}\varpi_k \frac{j\omega\left[m\omega^2 Y_0 \int_{x_1}^{x_2} w_k(x)dx - \gamma_k V_0\right]e^{j\omega t}}{\omega_k^2 - \omega^2 + 2j\zeta_k\omega_k\omega} \tag{25}$$

Accordingly, the electromechanical frequency response function (FRF) that relates the voltage output and vibration response to translational base accelerations is given by:

$$\frac{v(t)}{\omega^2 Y_0 e^{j\omega t}} = \frac{\sum_{k=1}^{\infty}\dfrac{jm\omega\varpi_k\left(\int_{x_1}^{x_2} w_k(x)dx\right)}{\omega_k^2 - \omega^2 + 2j\zeta_k\omega_k\omega}}{\left(\sum_{k=1}^{\infty}\dfrac{j\omega\gamma_k\varpi_k}{\omega_k^2 - \omega^2 + 2j\zeta_k\omega_k\omega}\right) + \dfrac{1+j\omega\tau_c}{\tau_c}} \tag{26}$$

3. Experimental procedure

3.1. *Preparation of EAPap piezoelectric film (EAPap piezofilm)*

Regenerated cellulose is a smart material that can be used for many applications [20]. It also exhibits strong piezoelectric effect and therefore can be used in energy transduction applications [5]. In addition, EAPap film has a good reversibility characteristic under bending strain and displacement which is suitable for such application. In this investigation, a thin regenerated EAPap cellulose piezoelectric film or simply EAPap piezofilm was used as the piezoelectric material which was prepared in the following process: Cellulose solution preparation, tape casting, washing, stretching and drying [6]. This process was well explained in previous articles [21,22] but is explained briefly for clarity.

EAPap piezofilm was prepared from raw cotton with 4500 degree of polymerization. The cotton that was cut into smallest pieces possible, and dissolved together with lithium chloride (LiCl) in anhydrous N, N-dimethyl acetamide (DMCAc) (Sigma Aldrich) with specific ratio at 110°C. To produce homogeneous solution, the mixture was continuously stirred using magnetic bar stirrer until LiCl and the raw cotton palp completely dissolved. To further ensure its homogeneity, it was centrifuged until transparent and highly viscous solution was produced. It was then poured on a clean glass plate and casted uniformly using doctor blade to produce a thin film. To remove the residuals of Li+(DMAc)x microcations, the film was washed with deionized water and isopropyl alcohol (IPA). While it is wet, the cellulose was stretched at 1.6 ratios from its original length which was subsequently dried under infrared light for approximately one hour. At this point, a thin regenerated cellulose film of 1.5×10^{-5} m thickness is produced with piezoelectric property. In this investigation, the thin film was cut at 45 degree relative to the stretching direction (see Figure 2(a)) with the length and width of 8 cm and 5 cm, respectively. For measurement purpose, aluminum electrodes were deposited on both sides of the film using thermal evaporating method. Finally, the EAPap film was
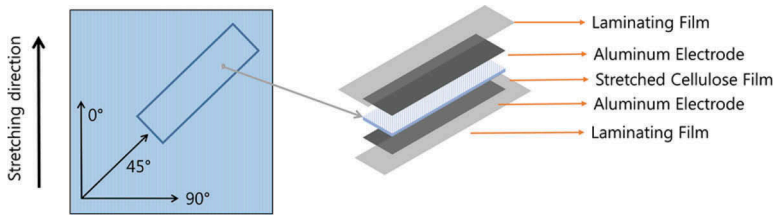
Figure 2. Schematic diagram showing the preparation of EAPap piezofilm. (a) Cutting orientation of the EAPap film. (b) Coating procedure of the EAPap film.

laminated with transparent films as shown in Figure 2(b). This is to prevent short circuit between the electrodes and the host as well as to reduce the possibility of damage.

It is notable that piezoelectric stress constant for EAPap film is 25 pC/N and material density and Young's modulus of the material are 1.4 kg/m3 and 3×10^9 N/m^2, respectively.

3.2. Preparation of the EAPap piezoelectric cantilever beam (EAPap piezobeam) energy harvester

Aluminum beam was used as a host structure in this investigation. It has the length of 200 mm, width of 50 mm and thickness of 1 mm. The beam's length is inclusive of 5 cm clamping area that has four holes for screw fastening, and the EAPap film with the dimensions of 80 mm length and 50 mm width attached 10 mm away from the clamp line as shown in Figure 3. The EAPap film was attached nearer to its clamped base where the largest bending was found, and this form EAPap piezoelectric cantilever beam, or simply EAPap piezobeam.

3.3. Measurement and calculation of current, voltage and power output of the piezobeam

The EAPap piezo beam prepared in the previous section was used as energy harvester in this investigation. It was fixed on the bobbin of an electromagnetic
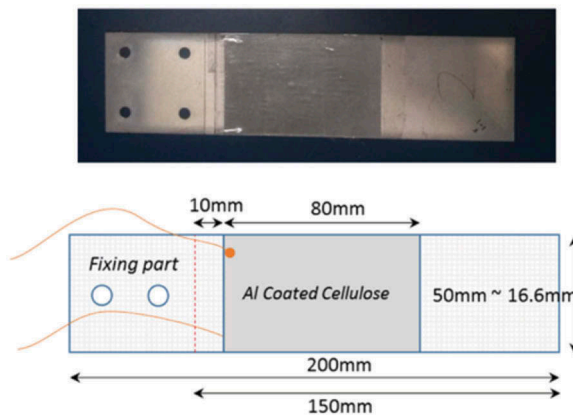
Figure 3. Photograph and schematic diagram of the EAPap piezobeam energy harvester.

shaker (Eliezer HEV-50) with tightening jig. 100 mV input voltage (corresponding to 2 mm of displacement input in this experiment) generated by a function generator (Agilent 33220A) and amplifier (Eliezer EA157) was used to excite the beam in the frequency range of interest. The experiment was carried out many times and every time before new measurement was taken, the input displacement was first determined using the accelerometer to make sure it is 2 mm. Otherwise, the input voltage, which is around 100 mV, is adjusted so that the displacement amplitude is maintained at 2 mm all the time. So, basically, it was the displacement that was controlled not the other parameters. To maintain the displacement input, an accelerometer was fixed at the fixed end of the EAPap beam to monitor its displacement and the input voltage was adjusted whenever necessary. The experimental arrangement is shown in Figure 4.

To provide varying external load, a potentiometer connected in series to the EAPap piezofilm was used. A picoammeter (Keithley 6485) and a pulse analyzer (Bruel & Kjaer 35360B-030) were used to measure the electric current in the circuit as well as the voltage output, respectively. The impedance of EAPap film was measured using LCR meter (HP 4282A) and with respect to the frequency changes.

4. Results, validation and discussion

4.1. *Resonance frequency of cellulose EAPap-based energy harvester*

Almost all of the mechanical energy harvesters are designed based on main resonance of the system and it is determined that matching vibration frequency and system's resonance frequency can lead to over one order of magnitude higher output power. Accordingly, finding the system resonance frequency is one of the most important points in power scavenger design process.

The resonance frequency of the system can be calculated using the theoretical equations developed in previous research works by submitting the substrate in
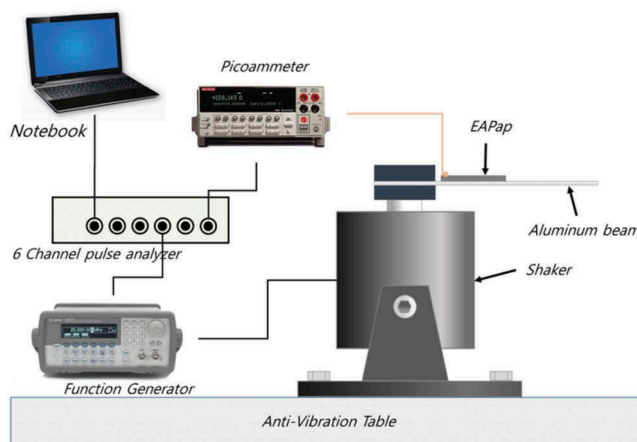
Figure 4. Experimental design and setup for the measurement of voltage and current output, and subsequently the harvested power from the excited EAPap piezobeam.

Equation (2) [13,14]. Substrate properties are as follows: Young's Modulus, E = 69 GPa, density of the beam, ρ = 2700 kgm^{-3}, thickness of the beam, $\square_s$ = 0.1 cm, width of the beam, w = 5 cm, and length of the beam, L = 15 cm.

The theoretical and experimental main resonance frequency of the EAPap-based energy harvester for the device is 34.6 Hz and 35.8 Hz, respectively. There is almost 3% error between the results that is negligible. Systematic and random errors can affect the results.

4.2. *EAPap energy harvester output*

In any energy scavenging circuit, for obtaining the highest value of output power, impedance matching of the external load to the energy harvester is unavoidable. For creating continuous change in impedances and therefore matching the source impedance (i.e. impedance of the cellulose EAPap films), a circuit including a potentiometer was used. The shaker excitation is a steady-state sinusoidal vibration in cantilever resonance frequency and therefore the output is a sinusoidal electric current. The impedance matching external load is 90 kΩ that is connected to the scavenging circuit.

Using a picoammeter, it was possible to measure the peak-to-peak current, I_{p-p}, that was 284 nA at resonance frequency of 35.8 Hz. Measuring the resistive load, R_{load}, and the root mean square (rms) current, I_{rms}, it is possible to calculate the mean power output, P_{mean}, as follows;

$$P_{mean} = I_{rms}{}^2 \times R_{load} = \left(0.354 \times I_{p-p}\right)^2 \times R_{load} \qquad (27)$$

The peak-to-peak current, I_{p-p}, and peak-to-peak voltage, V_{p-p}, was measured to be 284 nA and 25.6 mV, respectively. It is clear that $I_{p-p} = 2I_0$ and $V_{p-p} = 2V_0$. Also the maximum mean power output, P_{mean}, for the power scavenger was found to be 0.907 nW as the damping ratio of the structure was 0.006876 and resistance of the external load was 90 kΩ. As can be seen in Table 1, there is a good agreement between the experimental and theoretical values. So the relative error is negligible. So the theoretical method is validated and the method can be generalized for a range of damping ratios and resistive loads in different frequencies.

The theoretical output current, output voltage and mean power output was derived for damping ratio of 0.006876 and three different external loads $R_{L,1}$ = 90,000, $R_{L,2}$ = 128,000, $R_{L,3}$ = 150,000 for a wide range of frequencies between 0 rad/s to 1000 rad/s (about 0 Hz to 159.2 Hz) and the results can be seen in Figures 5–7 respectively. The results show that, the amount of scavenged energy around the resonance frequency is considerable and meaningful and the amount of load resistance R_L is an important parameter that shapes the system dynamic behavior. It is expected to have a short circuit condition when the load resistance is negligible ($R_L \rightarrow 0$). As can be expected for every excitation frequency, the maximum output

Table 1. Analytical and experimental results of the V_{p-p}, I_{p-p} and P_{mean} for partially covered cellulose-based piezoelectric energy harvester.

	I_{p-p} (nA)	V_{p-p} (mV)	P_{mean} (nW)
Theoretical values	295	26.6	0.9831
Empirical results	284	25.6	0.9071
Relative error (%)	3.9%	3.9%	8.4%
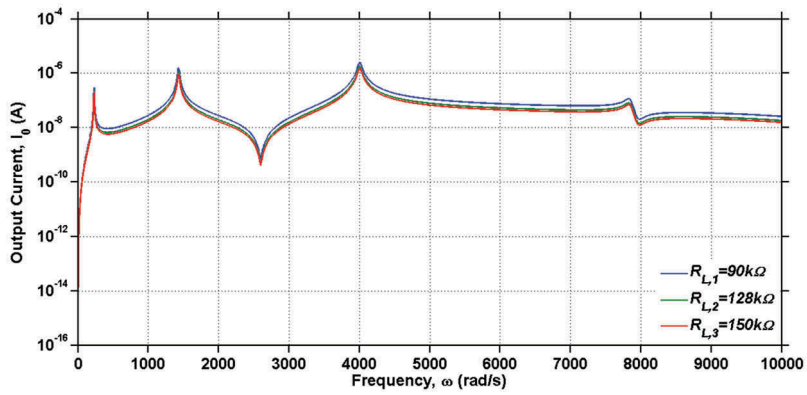
Figure 5. Maximum output current of the EAPap-based energy harvester for a range of frequencies with different external resistive loads.
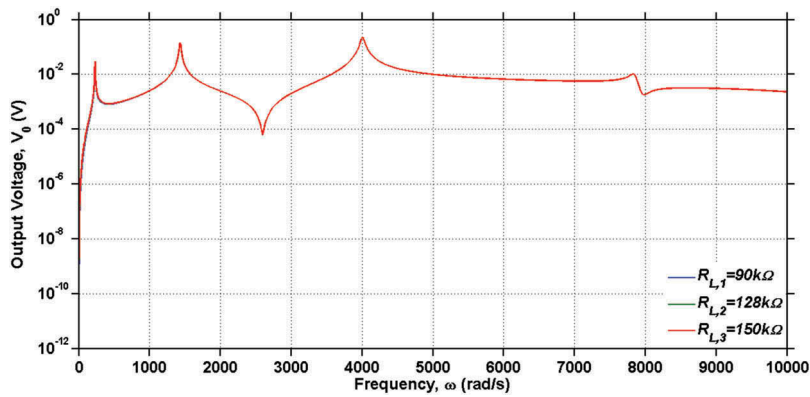
Figure 6. Maximum output voltage of the EAPap-based energy harvester for a range of frequencies with different external resistive loads.
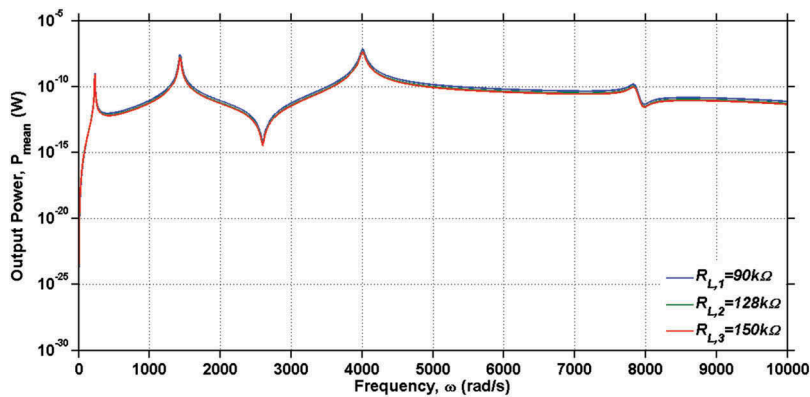

Figure 7. Mean power output of the EAPap-based energy harvester for a range of frequencies with different external resistive loads.

current is achieved when the system is close to short circuit conditions. As the same way, the large values of load resistances ($R_L \rightarrow \infty$), lead to open circuit behavior. Also, the highest output voltage can be attained for open circuit condition. As can be seen increasing the load resistance can lead to decreasing the output current and increasing the output voltage. Although the power output is simply the product of the current and the voltage, its behavior is more complicated than the current and the voltage. This behavior is due to the opposite behavior of current and voltage versus the resistive loads. So the power versus load resistance behavior is not monotonic [10], and it can be seen an optimal point in the graph. In this case study, maximum output power and so more scavenged power can obtained by choosing 90kΩ resistance.

To produce maximum electrical power from EAPap piezoelectric element which is attached to vibrating structure, the cantilever should be excited at its first natural frequency (228 rad/s) where it experiences the largest deflection. However the output voltage, current and the power are dependent on load resistance. By changing the load resistance at a certain frequency, the outputs are changed. Current, voltage and output power variations versus load resistance are shown in Figures 8–10, respectively. It is obvious that the voltage starts increasing with further increase in load resistance, while the current starts decreasing with further increase in load resistance and the previous results are confirmed. About the output power, there is an optimum point for which the highest output power is obtained. As can be seen there is a relative error between the experimental and theoretical value of optimum load resistance. In other words, the optimum load resistance, revealed that the energy loss in a piezoelectric cantilever with a given load resistance reaches its lowest possible amount. Accordingly the results show that when the vibration frequency is around the resonance frequency band of the piezoelectric energy harvester, the resistive load matching is an acceptable compromise for conditioning circuit design.

5. Summary and conclusion

Typically, a power harvester beam that is partially or fully covered by a piezoelectric film layer is located on a vibrating host structure and the scavenger cantilever generates
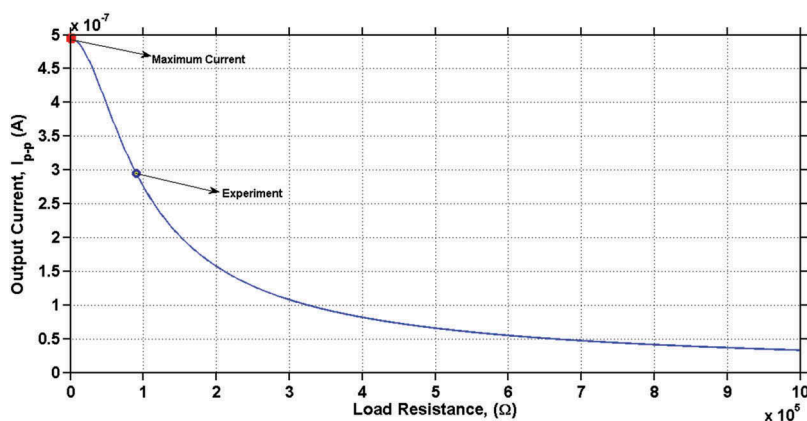
Figure 8. Peak-to-peak Output current versus different load resistances in a certain frequency (228 rad/s).
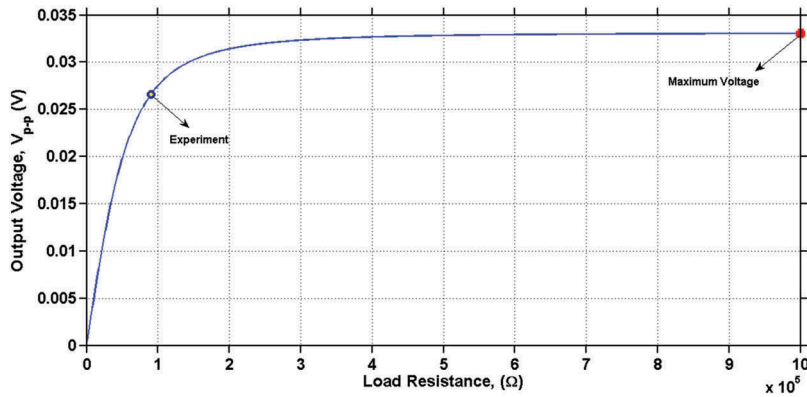
Figure 9. Peak-to-peak Output voltage versus different load resistances in a certain frequency (228 rad/s).

Figure 10. Mean output power versus different load resistances in a certain frequency (228 rad/s).

electrical energy due to base excitation. In this paper the closed-form analytical solution for a cantilever power scavenger that is partially covered by piezoelectric layer based on Euler-Bernoulli cantilever beam assumption is presented. Partially covered piezoelectric power scavengers are common devices that are used for generating energy which so far have not been studied in detail. The damping effect is considered in the modeling and the analytically extracted relations are then used in a parametric case study with a newfound piezo-polymer that has attracted the attention of many researchers and is known EAPap. The most common applications of this type of material are in actuators and sensors and a typical characteristic property of an EAP is that they will undergo a large amount of deformation while sustaining large forces. The results are validated using experiment and then output voltage, current and the power output are plotted against frequency for different external resistive loads. By applying harmonic excitation to a piezoelectric beam, it was shown that output power has a peak value within the load resistance range while the excitation frequency was remained constant. Yet, obtained voltage increases by increasing external load resistance and the electric current has an opposite behavior. So it can be seen that in

partially covered piezoelectric energy harvesters, there is an optimum amount for load resistance that leads to highest output power.

Disclosure statement

No potential conflict of interest was reported by the authors.

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