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Fixed-points theory for global vibration control using vibration neutralizer

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Abstract

The vibration neutralizer has been used in many applications since invented. In many cases, an ingenious design law called fixed-points theory was utilized in determining the optimum tuning and damping ratios of the device. However, those applications are limited to point response control of a relatively simple structure. There are some applications related to continuous structures but the purpose is for point response control, collocated or non-collocated. In this paper, the fixed-points theory is examined for global vibration control namely the control of the kinetic energy of a continuous structure. It is proven in this paper that the same design law is applicable for a more complicated purpose. The results presented in this paper may offer new ways of using the device.

1. Introduction

Historically, an auxiliary mass–spring–damper control system attached to a primary structure was known as ***vibration absorber*** or ***dynamic vibration absorber***. Perhaps the reason was because in the very early stage of its application, the device was designed to control vibration at a problematic resonance frequency of the structure. In this case, the vibration absorber acts as a tuned damper where its purpose is to dampen the motion of the structure at the resonance frequency of interest. However, there is a possible mode of operation where no damping exists in the device and it cannot absorb energy. So, the term ***absorber*** can be misleading [1]. For this reason, the term ***vibration neutralizer*** was recommended by Crede [2], and this terminology is adopted in this paper.

The vibration neutralizer was invented by Frahm [3]. Since then, the device has been extensively used to mitigate vibrations in various types of mechanical systems. The examples cited by Newland [4] and Steffen and Rade [5] are only a few to mention. In many cases, the vibration neutralizer was used to suppress the displacement amplitude of a structure that can be modeled as a simple system. Although the applications cited by Newland [4] and Steffen and Rade [5] are basically on continuous structures but the purpose is still for the control of some troublesome points or regions.

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The vibration neutralizer can be used is two distinct ways—to suppress the point vibration, collocated or non-collocated to the neutralizer, at the troublesome resonance frequency or at the troublesome frequency away from the resonance. In the former case, the simplest method of tuning the neutralizer is by making its resonance frequency coincides with the resonance of the primary structure. However, the neutralizer can become less effective, and may even increase the vibration of the primary structure when there is a change in the frequency of the primary force. The increase can be clearly seen on each side of the operating frequency in a frequency response function graph. In order to overcome this problem, an ingenious optimization method known as fixed-points theory was suggested by a proper selection of the neutralizer's stiffness (hence the natural frequency). The stiffness is chosen so that the heights of the two crossing points (or the fixed points) in the frequency response function become equal. This optimization technique is described in detail by den Hartog [6] and was believed to be originally suggested by Erich Hahnkamm [7,8]. The optimum tuning ratio of the neutralizer was found as a function of the neutralizer's mass given by

$$f_{\mathrm{opt}} = \frac{1}{1+\mu}, \tag{1}$$

where μ is the ratio of the neutralizer's to the primary structure's mass and $f_{\mathrm{opt}} = \omega/\omega_{\mathrm{opt}}$, ω being the forcing frequency and ω_{opt} is the optimal resonance frequency of the neutralizer. Later, Brock found that the damping in the neutralizer can also be optimized [9]. This was accomplished by making the amplitude of the two crossing points maximum. The optimum damping ratio in the neutralizer was also found as a function of the neutralizer's mass given by

$$\zeta_{\mathrm{opt}} = \sqrt{\frac{3\mu}{8(1+\mu)^3}}. \tag{2}$$

Since then, the fixed-points theory has become one of the design laws used in fabricating a neutralizer for the control of vibration of a relatively simple system.

The fixed-points theory has also been successfully used for the control of a point response of a continuous structure with well separated natural frequencies. The procedure on how to achieve this is summarized in [5]. However, it is well known that for continuous structure, reducing the vibration amplitude at a point may increase the amplitude at some other points [10]. This is true unless a global measure is taken where the aim is to reduce the overall vibrations of a structure, not only at a particular point. In this paper, the fixed-points theory is developed so that it can be used as a simple design law for global vibration control of a continuous structure.

2. Basic analytical formulation

In this section, the general formulation to describe the dynamic behavior of a continuous structure with J number of neutralizers attached is derived in terms of the vector of its transverse modal amplitudes. The system considered has a general configuration with arbitrary boundary conditions, and is forced by a point primary force, f_p. The derivations can be found in many articles but is briefly described here for convenience.

In general, the transverse displacement amplitudes of such a structure at any point can be expressed in terms of a finite number of modes as

$$w(x) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{q}, \tag{3}$$

where $\mathbf{\Phi}$ and $\mathbf{q}$ are the $M \times 1$ vector of the normalized mode shapes evaluated at point x and the $M \times 1$ vector of the modal displacement amplitudes of the structure, respectively. The $e^{i\omega t}$ dependence is assumed but is not shown for clarity. Without neutralizer, the modal displacement amplitudes can be written as

$$\mathbf{q} = \mathbf{A}\mathbf{g}_p, \tag{4}$$

where $\mathbf{A}$ and $\mathbf{g}_p$ are the $M \times M$ diagonal matrix of the complex modal amplitudes of the structure and the $M \times 1$ vector of the generalized primary force acting on the structure, respectively. $\mathbf{g}_p$ is given by $\boldsymbol{\Phi}(x_f)$F where x_f is the location of the excitation force and F is its amplitudes. The mth component of the complex modal amplitudes is given by

$$A_m = \frac{1}{M_s(\omega_m^2 - \omega^2 + \mathrm{i}2\zeta_s\omega\omega_m)}, \tag{5}$$

where M_s, ω_m and ζ_s are the total mass, the mth circular natural frequency and the damping ratio of the structure, respectively, and ω is the circular frequency of the excitation force. i is the imaginary number given by $\sqrt{-1}$.

When J number of neutralizers are fitted, the dynamic behavior of the structure is modified and become a coupled system. The modal displacement amplitudes of the new system is [11,12]

$$\mathbf{q} = [\mathbf{I} + \mathbf{A\Psi K\Psi}^{\mathrm{T}}]^{-1}\mathbf{Ag}_p, \tag{6}$$

where $\mathbf{I}$, $\mathbf{K}$ and $\mathbf{\Psi}$ are the M-length identity matrix, the J-length diagonal matrix of the dynamic stiffness of the neutralizers and the $M \times J$ matrix of the normalized mode shapes of the structure where the mjth entry is the modal amplitudes of the structure at the jth neutralizer's location. The dynamic stiffness of the jth neutralizer is [13]

$$K_j = -M_j\omega^2\left[\frac{1 + \mathrm{i}2\zeta_j\alpha_j}{1 - \alpha_j^2 + \mathrm{i}2\zeta_j\alpha_j}\right], \tag{7}$$

where M_j, ζ_j and α_j are the mass, the damping ratio and the tuning ratio of the jth neutralizer, respectively. The damping ratio is given by $\zeta_j = C_j/(2M_j\omega_j)$ where C_j is the damping coefficient of the jth neutralizer whereas the neutralizer's tuning ratio $\alpha_j = \omega/\omega_j$ where ω_j is the natural frequency of the neutralizer given by $\sqrt{k_j/M_j}$, and k_j and M_j are the neutralizer's stiffness constant and mass, respectively.

The time-averaged kinetic energy is taken as the measure of global vibration of the structure which is proportional to the sum of the squares of the modal velocity amplitudes. Mathematically, the kinetic energy is given by [14]

$$\mathrm{KE} = \frac{M_s\omega^2}{4}\mathbf{q}^{\mathrm{H}}\mathbf{q}, \tag{8}$$

where the superscript $^{\mathrm{H}}$ denotes the Hermitian transpose.

3. Development of the fixed-points theory for global vibration control

Consider there is only one neutralizer attached on the structure. In this case, Eq. (6) can be simplified to [11,12]

$$\mathbf{q} = [\mathbf{I} + K_k\mathbf{A\Phi}(x_k)\mathbf{\Phi}(x_k)^{\mathrm{T}}]^{-1}\mathbf{Ag}_p, \tag{9}$$

where K_k, x_k and $\Phi(x_k)$ are the dynamic stiffness of the neutralizer, the neutralizer's location on the structure and the modal amplitudes of the structure at the neutralizer's location, respectively. It should be noted that the neutralizer's index j in Eq. (9) has been replaced with k because there is only one neutralizer considered in the discussion. For a structure with well separated natural frequencies, the modal displacement amplitudes in the vicinity of the mth natural frequency can be well approximated by [11]

$$q_m = \frac{g_{pm}}{A_m^{-1} + K_k\phi_m^2(x_k)}, \tag{10}$$

where g_{pm} is the mth component of the generalized primary force. Structure such as beams is one of the good examples where natural frequencies are well separated. Therefore, in this paper a simply supported beam is considered in the discussions.

To reduce the mathematical burdens, the damping in the simply supported beam is set to zero and Eqs. (5) and (7) are written as

$$A_m = \frac{1}{(k_m - M_s\omega^2)}, \tag{11}$$

$$K_k = -M_k\omega^2\left[\frac{k_k + i2\zeta_k\sqrt{M_k k_k}\omega}{k_k - M_k\omega^2 + i2\zeta_k\sqrt{M_k k_k}\omega}\right], \tag{12}$$

respectively. k_m in Eq. (11) is the mth effective bending stiffness of the beam given by

$$k_m = (m\pi)^4\left(\frac{EI}{L^3}\right), \tag{13}$$

where E, I and L are the Young's modulus, moment inertia and the length of the beam, respectively. Therefore, Eq. (10) becomes, after rearrangement

$$q_m = \frac{g_{pm}[(i2\zeta_k\sqrt{M_k k_k}\omega) + \{k_k - M_k\omega^2\}]}{\begin{array}{c}[(i2\zeta_k\sqrt{M_k k_k}\omega)\{k_m - M_s\omega^2 - M_k\omega^2\phi_m^2(x_k)\} \\ +\{(k_m - M_s\omega^2)(k_k - M_k\omega^2) - M_k k_k\omega^2\phi_m^2(x_k)\}]\end{array}}. \tag{14}$$

The kinetic energy of the beam in the vicinity of the mth natural frequency can be written as [10]

$$\mathrm{KE}_m = \frac{M_s\omega^2}{4}\, q_m * q_m, \tag{15}$$

where the * denotes the complex conjugate. By taking

$$\begin{aligned}\mu &= M_k/M_s,\\ f_m &= \omega_k/\omega_m,\\ g_m &= \omega/\omega_m\end{aligned} \tag{16}$$

and substituting Eq. (14) into (15) gives, after rearrangement

$$\gamma_m = \left(\frac{A^2\zeta_k^2 + B^2}{C^2\zeta_k^2 + D^2}\right), \tag{17}$$

where

$$\begin{aligned}\gamma_m &= KE_m\frac{4m^4\pi^4 EI}{M_s L^3\omega^2 g_{pm}^2},\\ A &= 2f_m g_m,\\ B &= g_m^2 - f_m^2,\\ C &= 2f_m g_m\{g_m^2 - 1 + \mu g_m^2\phi_m^2(x_k)\},\\ D &= \mu f_m^2 g_m^2\phi_m^2(x_k) - (g_m^2 - 1)(g_m^2 - f_m^2).\end{aligned} \tag{18}$$

At this point, one should have noticed the difference in the definition of the damping ratio ζ_k given in [6] which may has been written as $\zeta_k = C_k/(2M_k\omega_m)$ if the same primary structure is considered whereas in this paper it is defined as $\zeta_k = C_k/(2M_k\omega_k)$ where C_k is the damping coefficient of the neutralizer. Because of that, the complete procedure on how to reach Eq. (17) is given in Appendix A.

The two fixed-points can be established by considering two cases—kinetic energy of the structure when the neutralizer's damping ratio is zero and when it is infinity. This is shown in Fig. 1 where the two points, P and Q are the common points to all curves regardless of the damping in the neutralizer. These points, P and Q are the fixed points for the kinetic energy of the structure with neutralizer attached. The physical properties of the primary structure are given in Section 4.

Using Eq. (17), these two cases can be expressed mathematically as

$$\gamma_m|_{\zeta_k=0} = \left(\frac{B^2}{D^2}\right), \tag{19}$$

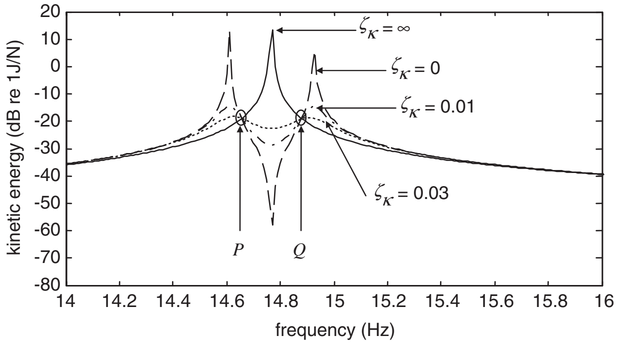

Fig. 1. Figure showing the existences of the fixed points P and Q for the kinetic energy of the beam. The figure is plotted in the vicinity of the first natural frequency and a neutralizer is attached at $x_k = 0.16L$ where four different neutralizer's damping ratios are shown with $\mu = 0.001$.

$$\gamma_m|_{\zeta_k=\infty} = \left(\frac{A^2}{C^2}\right), \tag{20}$$

respectively. The condition $(B/D)^2 = (A/C)^2$ implies the two crossing points of curves, $\mathrm{KE}|_{\zeta_k=0}$ and $\mathrm{KE}|_{\zeta_k=\infty}$, the fixed points we are seeking. Using the crossing point condition, it can be written that

$$\left\{\frac{g_m^2 - f_m^2}{\mu f_m^2 g_m^2 \phi_m^2(x_k) - (g_m^2 - 1)(g_m^2 - f_m^2)}\right\}^2 = \left\{\frac{1}{g_m^2 - 1 + \mu g_m^2 \phi_m^2(x_k)}\right\}^2. \tag{21}$$

Eq. (21) is similar to that found in [6], but with additional terms which is $\phi_m^2(x_k)$, the function of the neutralizer's location on the structure. Therefore, the same procedure described in [6] can be followed to define the optimum value of f_m in terms of the neutralizer's mass ratio μ. The procedure is briefly described here for convenience.

Eq. (21) can be reduced to a simpler form by taking its square roots but a $\pm$ve sign must be added to the right-hand side of the equation. The $-$ve sign is the trivial solution, therefore the fixed-points equation is given by

$$\frac{g_m^2 - f_m^2}{\mu f_m^2 g_m^2 \phi_m^2(x_k) - (g_m^2 - 1)(g_m^2 - f_m^2)} = \frac{1}{g_m^2 - 1 + \mu g_m^2 \phi_m^2(x_k)}. \tag{22}$$

Cross-multiplication and rearranging the resultant equation yields

$$g_m^4 - 2g_m^2\left\{\frac{1 + f_m^2 + \mu f_m^2 \phi_m^2(x_k)}{2 + \mu\phi_m^2(x_k)}\right\} + f_m^2\left\{\frac{2}{2 + \mu\phi_m^2(x_k)}\right\} = 0. \tag{23}$$

Supposed g_{m1}^2 and g_{m2}^2 are the roots of this equation, then

$$(g_m^2 - g_{m1}^2)(g_m^2 - g_{m2}^2) = g_m^4 - (g_{m1}^2 + g_{m2}^2)g_m^2 + g_{m1}^2 g_{m2}^2 = 0. \tag{24}$$

Comparing Eqs. (23) and (24), one obtains

$$g_{m1}^2 + g_{m2}^2 = 2\left\{\frac{1 + f_m^2 + \mu f_m^2 \phi_m^2(x_k)}{2 + \mu\phi_m^2(x_k)}\right\}. \tag{25}$$

According to the fixed-points theory, the kinetic energy at those two roots must be equal and independent of the damping in the neutralizer. This occurs when either Eqs. (19) or (20) is satisfied. For simplification, Eq. (20) is used and substituting the two roots,

$$\gamma_m\Big|_{\zeta_k=\infty} = \left\{\frac{1}{g_{m1}g_{m2}(1 + \mu\phi_m^2(x_k)) - 1}\right\}^2 \tag{26}$$

or

$$\sqrt{\gamma_m\big|_{\zeta_k=\infty}} = \pm\frac{1}{g_{m1}g_{m2}(1+\mu\phi_m^2(x_k))-1}. \tag{27}$$

However, the two equations in Eq. (27) must be the same based on the fixed-points theory and thus

$$\frac{1}{g_{m1}^2(1+\mu\phi_m^2(x_k))-1} = \frac{-1}{g_{m2}^2(1+\mu\phi_m^2(x_k))-1} \tag{28}$$

or

$$g_{m1}^2 + g_{m2}^2 = \frac{2}{1+\mu\phi_m^2(x_k)}. \tag{29}$$

From Eqs. (25) and (29), the optimum tuning condition is obtained when

$$f_{m\mathrm{opt}} = \frac{1}{1+\mu\phi_m^2(x_k)}. \tag{30}$$

This has a similar form with the optimum tuning condition for the single degree of freedom (SDOF) primary system in Eq. (1) except the additional terms which is the modal amplitudes of the structure at the neutralizer's location, $\phi_m^2(x_k)$.

It has been proven that it is possible to get the optimum tuning of a neutralizer for global vibration control of a continuous structure. The procedure is similar to that found in [6] to suppress the displacement amplitudes of an undamped SDOF system. It can also be proven that the optimum damping of the neutralizer can be found using the similar approach suggested by den Hartog [6] and Brock [9]. However, the procedure is long and tedious and therefore is not shown here. The optimum damping of the neutralizer for global control of a continuous structure has a similar form as for the displacement of an SDOF system except again the new terms which is $\phi_m^2(x_k)$. In complete, the optimum damping of the neutralizer can be proven as

$$\zeta_{k\mathrm{opt}} = \sqrt{\frac{3\mu\phi_m^2(x_k)}{8(1+\mu\phi_m^2(x_k))}}. \tag{31}$$

Again, it should be noted the difference between the expression of the optimum damping ratio given in Eq. (31) and the one found in [6,9]. This is the result from the difference in the definition of the damping ratio discussed earlier. The two expression ζ (as found in [6,9]) and ζ_k can be linked by an expression $\zeta = f_m\zeta_k$ where f_m is defined in Eq. (16). Therefore, $\zeta_{\mathrm{opt}} = f_{m\mathrm{opt}}\zeta_{k\mathrm{opt}}$. Substituting Eqs. (30) and (31) gives

$$\zeta_{k\mathrm{opt}} = \left\{\frac{1}{1+\mu\phi_m^2(x_k))}\right\}\sqrt{\frac{3\mu\phi_m^2(x_k)}{8(1+\mu\phi_m^2(x_k))}} = \sqrt{\frac{3\mu\phi_m^2(x_k)}{8(1+\mu\phi_m^2(x_k))^3}},$$

which gives the same forms optimum damping as found in [6,9].

4. Simulation results and discussion

To facilitate a better understanding on the investigation in this paper, simulation results are presented in this section on the effects of the optimum neutralizer to the kinetic energy of the beam. The investigation is carried out on the first three natural frequencies of the beam so that a general conclusion can be made. As has been stated earlier, for global vibration control, the fixed-points theory may only be applicable to a continuous structure with well-separated natural frequencies. For this reason, a simply supported beam with the following properties was selected to be used in the numerical simulations: Physical dimensions $= 1\,\mathrm{m}\times 0.0381\,\mathrm{m}\times 0.00635\,\mathrm{m}$; material density $= 7870\,\mathrm{kg/m^3}$; Young's modulus $= 207\mathrm{E}9\,\mathrm{N/m^3}$; damping ratio $= 0.005$ and unity amplitude of a primary point force is applied at $0.1L$ ($x_f = 0.1L$). Fig. 2 shows the total kinetic energy of the beam contributed from the first 10 modes, in comparisons with the kinetic energy of each of the contributing mode. It is clearly seen that over a narrow band in the vicinity of each

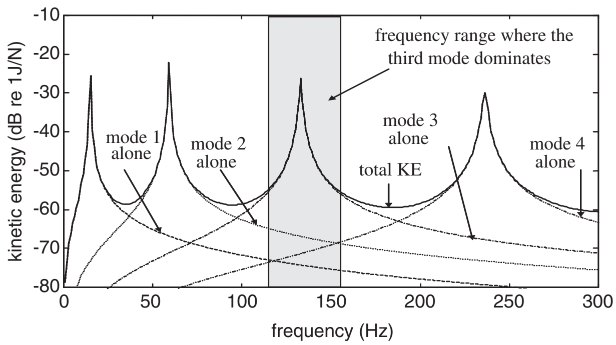

Fig. 2. Comparison between the total kinetic energy of the beam and the kinetic energy of each of the contributing modes [10].

natural frequency, the kinetic energy is mainly dominated by the related mode number. Therefore, it is anticipated that the optimization can be used for the above beam.

There are two cases considered in this investigation:

1. The effects of the optimum neutralizer with different mass ratios and
2. the effects of the positioning of the optimum neutralizer on the beam.

It should be noted that all simulations in the following discussion used ten modes in the calculation of the kinetic energy.

4.1. Effects on the kinetic energy with different neutralizer's mass ratios

Fig. 3 shows the first test for the fixed-points theory for global vibration control. The control target is the first natural frequency and the neutralizer is applied at $x_k = 0.5L$ where the modal amplitude $\phi_m(x_k)$ has the highest value. There are five curves shown to show the effects of changing the value of μ with uncontrolled curve as a reference. It can be seen in Fig. 3(a) that the kinetic energy curves are relatively flat for all μ and this is clearly shown in Fig. 3(b), the close up of the kinetic energy curves in Fig. 3(a) in the vicinity of the first natural frequency. Similar results are observed for the second and third resonance frequency shown in Figs. 4 and 5, respectively. In all cases, the kinetic energy in the vicinity of the respective natural frequency is reduced and no new resonance is observed. However, the most important observation is that all curves are reasonably smooth. This proves that the fixed-points theory works well on the beam with the selected properties.

As the mass ratio μ increased, the reductions in the kinetic energy also increased. When μ is high enough, it is possible to remove the resonance effects in the kinetic energy of the structure. For the third natural frequency (Fig. 5), the resonance effects can be removed when μ is 5%, a relatively small value. However, at a lower resonance frequency, it requires higher value of μ. This is well-understood phenomenon where a high neutralizer's mass is required to make it more effective in the lower frequency region.

It can be interesting to investigate the effect of the damping value in the host structure to the application of the fixed-points theory. This effect is illustrated in Fig. 6. It can be seen that as long as the damping value is small, the theory work well for global control purpose where the kinetic energy curves at different damping value are almost coincides with each other. Three different values of the damping ratio in the beam are presented in the simulation and they are 0, 0.005 and 0.01.

4.2. Effects on the kinetic energy when the neutralizer is applied at different location

The optimum tuning and damping ratios given in Eqs. (30) and (31), respectively, indicates the influence of the positioning of the neutralizer on the structure. This influence is shown in Fig. 7 with the first natural frequency as the control target and the neutralizer's location is varied. It can be seen that the kinetic energy is the lowest when the neutralizer is located at $x_k = 0.5L$, then followed by the location at $x_k = 0.35L$ and

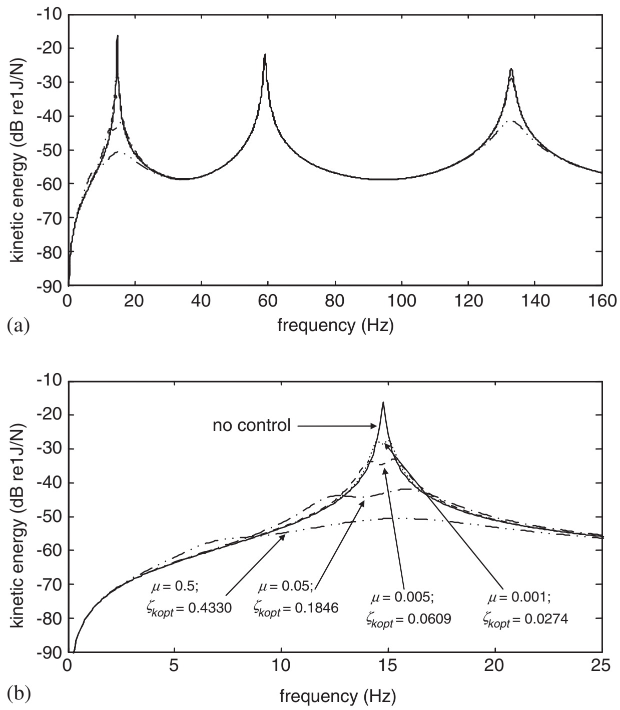

Fig. 3. Kinetic energy of the beam with optimum neutralizer fitted at $x_k = 0.5L$. Four different mass and optimum damping ratios are shown with first natural frequency as the control target: (a) figure showing all three modes; (b) in the vicinity of the first mode.

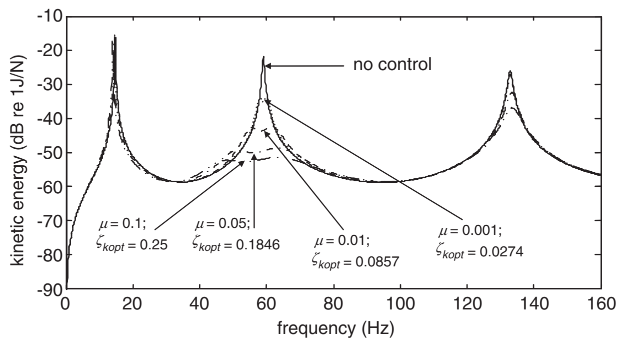

Fig. 4. Kinetic energy of the beam with optimum neutralizer fitted at $x_k = 0.25L$. Four different mass and optimum damping ratios are shown with second natural frequency as the control target.

$x_k = 0.25L$. This is because the modal amplitudes $\phi_m(x_k)$ has the highest value at $x_k = 0.5L$ compared to other locations. Similar result is seen in Fig. 8 when the control target is the second natural frequency. In other words, the achievable reductions in the kinetic energy become smaller as the neutralizer's location approaches a nodal point. A neutralizer located at a nodal point gives no effect to the kinetic energy of the structure at the targeted natural frequency.

As the neutralizer's location approaches a nodal point, the kinetic energy curve is no longer flat. This effect is shown in Fig. 9 where the curve at a lower frequency is located at a higher kinetic energy compared to the curve at the higher frequency. This effect could be interesting to investigate but is out of the scope of this paper.

Although in general, the reductions in the kinetic energy depends to the value of the modal amplitudes of the structure at the neutralizer's location, location with the same modal amplitudes does not necessarily results

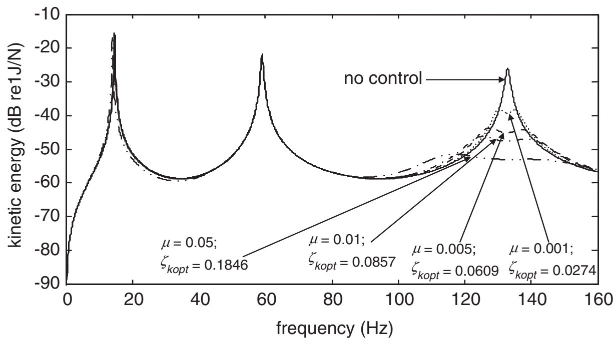

Fig. 5. Kinetic energy of the beam with optimum neutralizer fitted at $x_k = 0.5L$. Four different mass and optimum damping ratios are shown with third natural frequency as the control target.

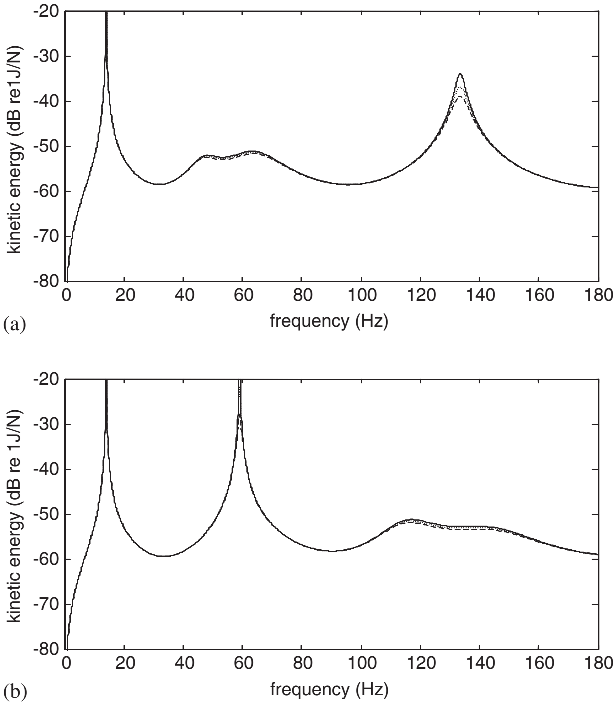

Fig. 6. Kinetic energy curves of the beam with optimum neutralizer attached where the beam has different damping values, ζ_s. Solid line—$\zeta_s = 0$; dotted line—$\zeta_s = 0.005$; dashed line—$\zeta_s = 0.01$: (a) control target is the second natural frequency. $\mu = 0.1$ and $x_k = 0.5L$; (b) control target is the third natural frequency and $\mu = 0.05$ and $x_k = 0.25L$.

in the same amount of reductions. This is clearly shown in Fig. 9 when the control target is the third natural frequency of the beam. It can be seen that although $|\phi_3(L/6)| = |\phi_3(L/2)| = |\phi_3(5L/6)|$ and the value of the optimum damping for three locations are the same but the kinetic energy is lower for the neutralizer located at $L/6$ followed by $L/2$ and $5L/6$. This suggests that the neutralizer's location relative to the point force also influences the amount of reduction in the kinetic energy.

5. Summary and conclusion

In this paper, the fixed-points theory has been developed and examined for global vibration control of a continuous structure using vibration neutralizer. The same procedure as in the conventional fixed-points

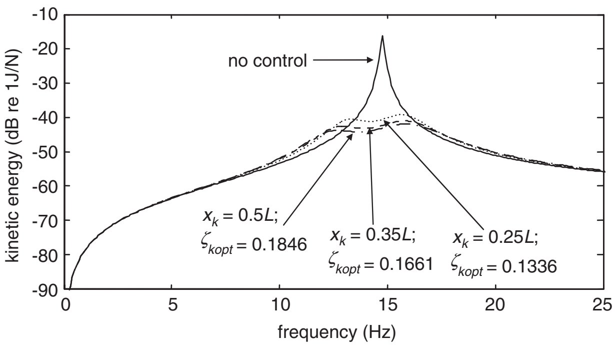

Fig. 7. Kinetic energy of the beam with optimum neutralizer but applied at three different locations one at a time. The control target is the first natural frequency and $\mu = 0.05$.

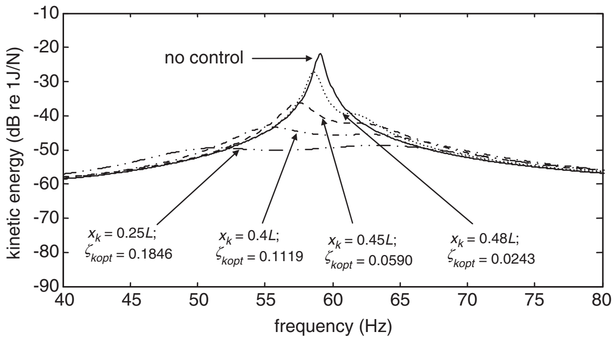

Fig. 8. Kinetic energy of the beam with optimum neutralizer but applied at four different locations one at a time. The control target is the second natural frequency and $\mu = 0.05$.

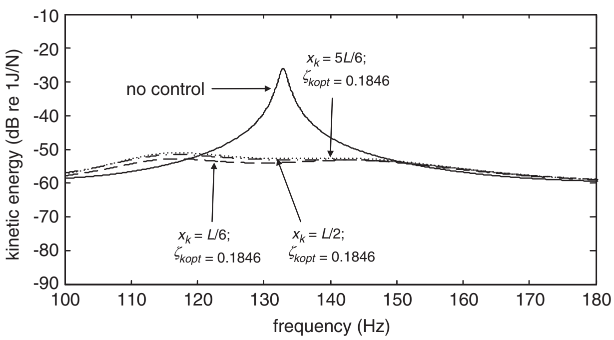

Fig. 9. Kinetic energy of the beam with optimum neutralizer but applied at three different locations one at a time. The control target is the third natural frequency and $\mu = 0.05$.

theory has been used to derive the optimum tuning and damping ratios of the device. It was found that the optimum tuning and damping ratios have similar form as in the conventional theory but with additional terms, which is the modal amplitude of the structure at the neutralizer's location. Using these optimum values, it is possible to remove the resonance effects in the structures kinetic energy at reasonable mass ratio. From the simulation results, it is recommended that the neutralizer is applied at a point where the structure has the highest deflection amplitude for the natural frequency concerned. The results presented in this paper may offer new ways of using the device over the conventional one. It can be used, for example, not only to reduce the vibration at a point of an actual structure such as bridges or buildings but also the vibration at all parts of the structure concerned.

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Appendix A

In this appendix, the procedure on how to reach Eq. (17) is provided. By substituting Eq. (14) into (15), the kinetic energy in the vicinity of the mth natural frequency can be written as

$$\mathrm{KE}_m = \frac{M_s\omega^2 g_{pm}^2}{4}\frac{[(2\zeta_k\sqrt{M_k k_k}\omega)^2 + \{k_k - M_k\omega^2\}^2]}{\begin{array}{c}[(2\zeta_k\sqrt{M_k k_k}\omega)^2\{k_m - M_s\omega^2 - M_k\omega^2\phi_m^2(x_k)\}^2 \\ +\{(k_m - M_s\omega^2)(k_k - M_k\omega^2) - M_k k_k\omega^2\phi_m^2(x_k)\}^2]\end{array}}. \tag{A.1}$$

If the equation is divided by $(M_k/M_k)^2$ then

$$\mathrm{KE}_m = \frac{M_s\omega^2 g_{pm}^2}{4}\frac{[(2\zeta_k\omega_k\omega)^2 + \{\omega_k^2 - \omega^2\}^2]}{\begin{array}{c}[(2\zeta_k\omega_k\omega)^2\{k_m - M_s\omega^2 - M_k\omega^2\phi_m^2(x_k)\}^2 \\ +\{(k_m - M_s\omega^2)(\omega_k^2 - \omega^2) - M_k\omega_k^2\omega^2\phi_m^2(x_k)\}^2]\end{array}}. \tag{A.2}$$

Eq. (A.2) can also be written as

$$\mathrm{KE}_m = \frac{M_s\omega^2 g_{pm}^2}{4M_s^2}\frac{[(2\zeta_k\omega_k\omega)^2 + \{\omega_k^2 - \omega^2\}^2]}{\begin{array}{c}[(2\zeta_k\omega_k\omega)^2\{\omega_m^2 - \omega^2 - \mu\omega^2\phi_m^2(x_k)\}^2 \\ +\{(\omega_m^2 - \omega^2)(\omega_k^2 - \omega^2) - \mu\omega_k^2\omega^2\phi_m^2(x_k)\}^2]\end{array}}, \tag{A.3}$$

Eq. (A.3) can be divided by $(\omega_m/\omega_m)^4$ to get

$$\mathrm{KE}_m = \frac{M_s\omega^2 g_{pm}^2}{4M_s^2}\frac{[(2\zeta_k f_m g_m)^2 + \{f_m^2 - g_m^2\}^2]}{\begin{array}{c}[(2\zeta_k f_m g_m)^2\{\omega_m^2 - \omega^2 - \mu\omega^2\phi_m^2(x_k)\}^2 \\ +\{(1 - g_m^2)(\omega_k^2 - \omega^2) - \mu f_m^2\omega^2\phi_m^2(x_k)\}^2]\end{array}}, \tag{A.4}$$

where f_m and g_m is Eq. (A.4) are given in Eq. (16). Dividing only the denominator with $\omega_m^4 = k_m^2/M_s^2$ yields

$$\mathrm{KE}_m = \frac{M_s\omega^2 g_{pm}^2}{4k_m^2}\frac{[(2\zeta_k f_m g_m)^2 + \{f_m^2 - g_m^2\}^2]}{\begin{array}{c}[(2\zeta_k f_m g_m)^2\{1 - g_m^2 - \mu g_m^2\phi_m^2(x_k)\}^2 \\ +\{(1 - g_m^2)(f_m^2 - g_m^2) - \mu f_m^2 g_m^2\phi_m^2(x_k)\}^2]\end{array}}. \tag{A.5}$$

Eq. (A.5) can be rearranged to get Eq. (17).

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