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# The effects of width reduction on the damping of a cantilever beam and its application in increasing the harvesting power of piezoelectric energy harvester

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**Abstract**

Previous work shows that when a cantilever piezoelectric energy harvester with a given width is split into several pieces and then electrically connected in parallel, the output power increases substantially compared with when it acts in a single piece with a similar total width. It was hypothesized that this increase is due to the reduction in the damping of the width-reduced beam. As a result, the beam with the smaller width vibrates with higher amplitudes and therefore has higher energy harvesting capability. In this paper, this hypothesis is examined by measuring the damping of the cantilever beam as its width is reduced. It is shown that as the width decreases, the damping is reduced, which contributes to the increase in the harvested power. It is then shown that the harvested energy from an array of cantilever piezobeams with a certain total width is higher than that from a single-piece harvester of similar width.

Keywords: piezoelectric energy harvester, ambient vibration, structural damping, energy conversion, MEMS, width-splitting method

(Some figures may appear in colour only in the online journal)

## 1. Introduction

In recent years, microenergy harvesting has been a focus of research with a view to providing autonomous operation for low-power electronic devices such as wireless sensors, implantable medical devices, and microelectromechanical systems (MEMSs). One appealing type of energy harvester is piezoelectric materials that have the capability to convert ambient vibration into electricity. Intensive research has been carried out to improve energy harvesting, in particular, to find ways to increase the harvested power from the harvester. This includes using different materials such as PVDF [1], electro-active polymer EAP [2], Nafion [3], and, more recently, cellulose-based electro-active paper [4]. Structural tailoring and modification of piezoelectric materials have also been looked into. The use of different designs such as cantilever beams [5] with proof mass [6], cymbals [7], stacks [8], plates [9], and shell structures [10] have been extensively studied. In addition, for energy harvesting, piezoelectric elements have also been used simultaneously as power generators and sensors [11–13].

Piezoelectric materials are commonly used in the form of a cantilever beam embedded in a system [14–24]. Hence, its structural setup is relatively simple and also provides the flexibility to alter its physical dimensions as well as its equivalent mass for the best harvesting performance. Quite

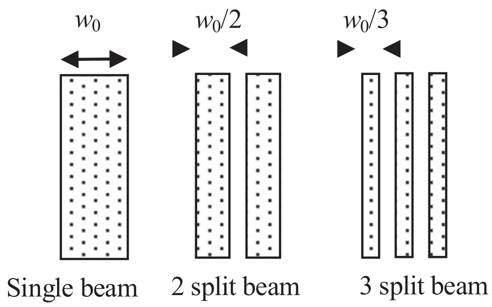



**Figure 1.** Piezoelectric energy harvester in the form of a single cantilever beam, two split beams, and three split beams. The total width of the beam or beams, $w_0$, remains the same.

recently, investigations showed that if a piezoelectric energy harvester in the form of a cantilever beam is split into several pieces (refer to figure 1) and then electrically connected in an array, the power output will become higher compared with the output of a single piezobeam of similar total width, length, and thickness [25, 26]. The two published papers used a specific method called folding where, for a given initial width, the piezoelectric energy harvester is folded and then split equally. The folding and splitting are continued for higher numbers of splits.

In previous investigations, it was reported that there is an increase of 45% in harvested power for single folding, whereas for double folding, the increase can be as high as 75% compared with no splitting. The increase in the harvested power was believed to be due to reduction in the damping ratio of the width-reduced beam. With a lower damping value, the beam vibrates with high amplitude, which implies that it harvests surrounding vibrational energy more efficiently. Combining such smaller beams in parallel results in a higher value of total harvested power compared with a single beam of similar total width. However, no supporting data was provided in the papers.

This paper is an extension of previous works. A brief theoretical background is first presented in section 2. Then an experimental investigation into the effects of width reduction of an aluminum cantilever beam is described in section 3. The increase in the vibration amplitudes of the width-reduced beams when they are excited with an external force is discussed in section 4. An example application from the findings in this paper is discussed in section 5, and conclusions are given in section 6.

## 2. A brief theoretical background of the width-splitting method

Width splitting was introduced to increase the harvested output power from a piezoelectric energy harvester in the form of a cantilever beam by splitting a given piezobeam into smaller elements of identical width. These smaller elements were then electrically connected in parallel to form an array of piezoelectric beams. It was hypothesized that because each individual piezobeam in the array had a smaller width, the damping was reduced. Therefore, the individual element vibrated at higher amplitudes, resulting in high voltage output. The governing principle of the width-splitting method is briefly discussed here for clarity. Details of the theory can be found in [25, 26].

For a single piezo composite where a piezofilm is attached to a host structure, the instantaneous open circuit voltage output can be written as

$$V = \frac{-3AB(1-A)g_{31}LKd_o}{hwt_b} \sqrt{\frac{(2\xi r)^2 + 1}{(2\xi r)^2 + \left(1 - r^2\right)^2}} \sin(2\pi ft) \tag{1}$$

when the piezo bender is under excitation from an external force. $A$ is the thickness ratio between the host structure and the piezofilm; $B$ is the Young's modulus ratio between the host structure and the piezofilm; $g_{31}$ is the stress constant of the piezofilm; $L$ is the length of the piezofilm; $K$ is the spring constant of the bender (the composite of the piezofilm and the host structure); $d_o$ is the base displacement amplitude, $h = 1 + A^4(1-B)^2 - 2A(2A^2 - 3\mathrm{A} + 2)(1 - \mathrm{B})$; $w$ is the width of the bender; $t_b$ is the thickness of the piezofilm; $\xi$ is the damping ratio of the bender; $r$ is the ratio between the excitation frequency and the resonance frequency of the bender; and $f$ is the frequency of the excitation force. When the piezo energy harvester is folded and split equally, the open circuit voltage output becomes

$$V = \frac{\dfrac{-3AB\left(2^N\right)(1-A)g_{31}LKd_o}{hwt_b}}{\sqrt{\dfrac{(2\xi r)^2 + 1}{(2\xi r)^2 + \left(1 - r^2\right)^2}} \sin(2\pi ft)}, \tag{2}$$

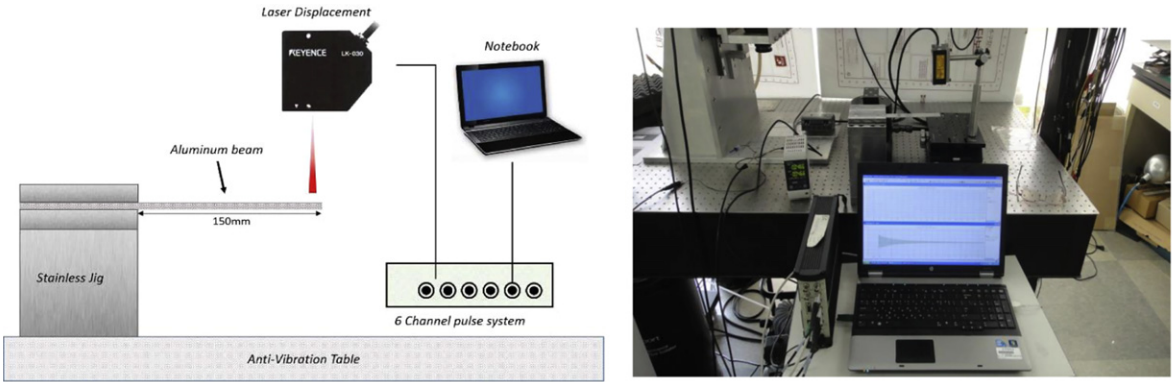
where $N$ is the number of folds and splits. The harvested root mean square (RMS) voltage across a load $R_{Load}$ is therefore written as

$$V_{Load,N,RMS} = \frac{-3AB\left(2^N\right)(1-A)g_{31}LKd_oR_{Load}}{hwt_b} \times \sqrt{\frac{(2\xi r)^2 + 1}{2\left(R_{Load}{}^2 + \left(2\pi fC_N\right)^{-2}\right)\left((2\xi r)^2 + \left(1 - r^2\right)^2\right)}} \times \sin(2\pi ft). \tag{3}$$

$C_N$ is the piezoelectric capacitance given by

$$C_N = \frac{(1 - A + AB)hw_oL\varepsilon_r\varepsilon_o}{(1 - A)t_bk}, \tag{4}$$

where $w_o$ is the initial width of the beam, $\varepsilon_r$ is the permittivity of the piezo material, and $\varepsilon_o$ is the permittivity of free space. It can be seen that theoretically, as the piezo bender is split




(a). Schematic diagram (b) Photo of the experimental setup

**Figure 2.** The experimental setup to measure the damping of a width-reduced cantilever beam. (a) Schematic diagram (b) Photograph of the experimental setup.

and then electrically connected in parallel to form an array of smaller piezobeams, the harvested voltage across the load will increase with the number of splits. The harvested power across the load is given as

$$P_N = \frac{{V_{Load,N,RMS}}^2}{R_{Load}}, \tag{5}$$

which also increases with the number of splits.

## 3. Effects of width reduction on the damping of a cantilever beam

This section describes the experimental procedure and the results of the main part of the paper, which is the investigation of the effects on damping in a beam when its width is reduced. The experiment was carried out using an aluminum beam 1 mm thick and 15 cm long, with an initial width of 5 cm. With these dimensions, the fundamental resonant frequency was calculated to be 36 Hz. One end of the beam was clamped onto an experimental rig, and the displacement at the free end was measured by using a laser displacement sensor when excited with an impulse force at the tip. Similarly, the displacement of the beam's tip was measured for another five beams of decreasing width. For all the beams used in the experiment, their first natural frequency was calculated to be the same as for the beam with the initial width of 5 cm, i.e., 36 Hz. Figure 2 shows the setup of the experiment.

The damping ratios and natural frequencies of the beams were determined using the logarithmic decay laws of the displacement amplitude and a fast Fourier transform (FFT) algorithm performed on the time history of the beam's displacement, respectively. Table 1 shows the damping ratios for all the beams in the experiment. It can be seen that as the width of the beam decreases, the damping ratios also decrease. To get a clearer prospective, the data in table 1 is plotted in figure 3. It can be seen that the damping decreases with the width. For example, for the smallest beam, with a width of 0.8 cm, the damping ratio decreased more than 50% compared with the beam with the initial width of 5 cm. It is worth mentioning here that the measured first resonance frequency of all the beams was around 36 Hz, with minor discrepancies due to changes in the damping value.

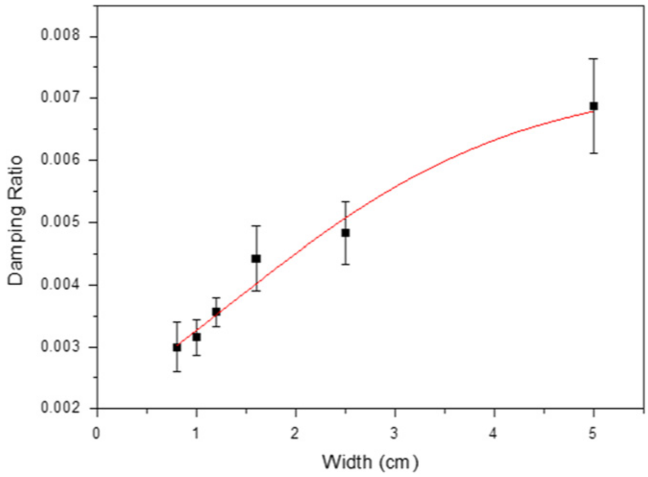


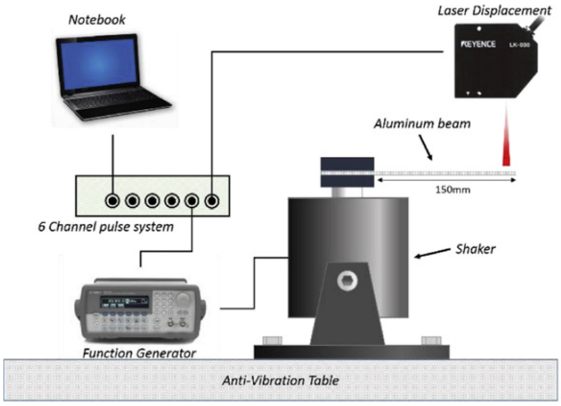

**Figure 3.** Graph showing the changes in the beam's damping ratio as the width is changed.

## 4. Effects of the beam's width reductions on its displacement amplitudes at the resonant frequency

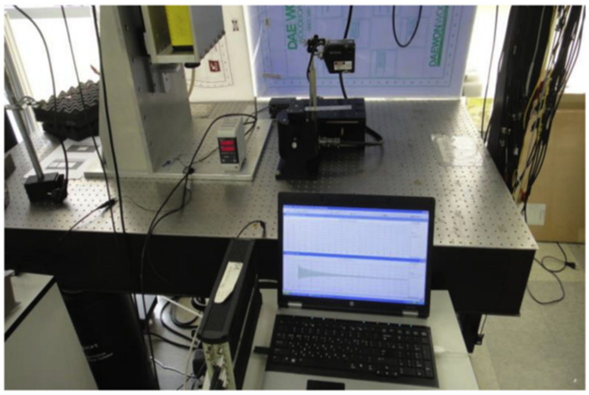
It was also hypothesized that since the damping of the width-reduced beam decreased, the displacement amplitudes at the resonant frequency would increase. This is because the input energy of the beam is less dissipated (because the damping has decreased), and therefore, most of the power is turned into vibrational energy. To prove this, the vibration amplitudes at the tips of all the beams in the preceding section were measured and compared with one another with external excitation at the clamp base. The tip vibration was measured by using a laser displacement sensor when one end of the beam was fixed on a shaker and excited with an input voltage of 100 mV RMS. Figure 4 shows the schematic diagram and a photograph of the experimental setup.

**Table 1.** Damping ratios of the aluminum cantilever beam at different widths.

| Width (cm) | 0.8 | 1.0 | 1.2 | 1.6 | 2.5 | 5 |
|---|---|---|---|---|---|---|
| Damping ratio | 0.002 995 | 0.003 155 | 0.003 559 | 0.004 424 | 0.004 822 | 0.006 876 |




(a). Schematic diagram



(b) Photo of the experimental setup

**Figure 4.** The experimental setup to measure the tip displacement of the width-reduced cantilever beam. The beam is excited with an external force by using a shaker at the clamped end with an input voltage of 100 mV RMS. (a) Schematic diagram (b) Photograph of the experimental setup.

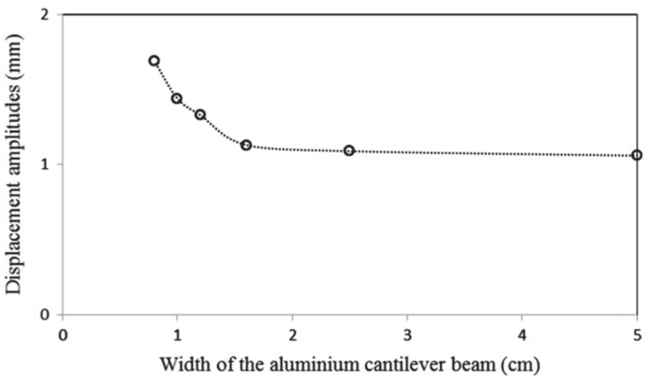



**Figure 5.** The peak-to-peak displacement amplitudes at the tip of the cantilever beam with different widths.

## 5. Increasing the harvesting power of a piezoelectric energy harvester by using an array of width-reduced beams

It has been proved that as the width of a beam is reduced, the damping decreases accordingly. As a consequence, the width-reduced beam vibrates at high amplitudes. This finding can be used to increase the power that is harvested by a piezoelectric energy harvester. To show this, four sets of copper–tin (CuSn) piezobeams were used with different numbers of splits. The CuSn piezobeam was made in cantilever form, where one end is clamped to a shaker whereas the end is free. The setup was similar to that in section 4 except that for the experiment in this section three sets were in the form of an array of a certain number of smaller piezobeams depending on the number of splits (see table 3). A proof mass was added at the tip of each beam so that they vibrated at the first resonant frequency of 70 Hz. The input voltage of the shaker was fixed at 500 mV RMS.

**Table 2.** Peak-to-peak vibration amplitudes at the tips of the beams as they are excited at the resonant frequency with an input voltage of 100 mV RMS for the sine wave at the clamped base.

| Beam width (cm) | 5 | 2.5 | 1.6 | 1.2 | 1.0 | 0.8 |
|---|---|---|---|---|---|---|
| Excitation frequency (at the resonant frequency) (Hz) | 32.5 | 33.9 | 34.3 | 34.4 | 35.1 | 35.3 |
| Peak-to-peak displacement amplitude (mm) | 1.06 | 1.09 | 1.13 | 1.33 | 1.44 | 1.69 |

Table 2 shows the vibrational amplitudes at the tips of all the beams, which are plotted against the beam's width in figure 5. It can be seen that the displacement amplitudes are always higher for smaller beams. The increase in the displacement amplitudes is as high as 60% for the 0.8 cm beam compared with the beam with the initial width (the reduction in width is 84%). Throughout the experiment, additional mass was added to the clamp area of the beam to account for the loss in weight of the width-reduced beam compared with the original value.

Table 3 shows information about the piezobeams used in the experiment, including their RMS voltage output and their power output across a 100 kΩ load. It can be seen that, in comparison with the single piezobeam, the harvested voltage

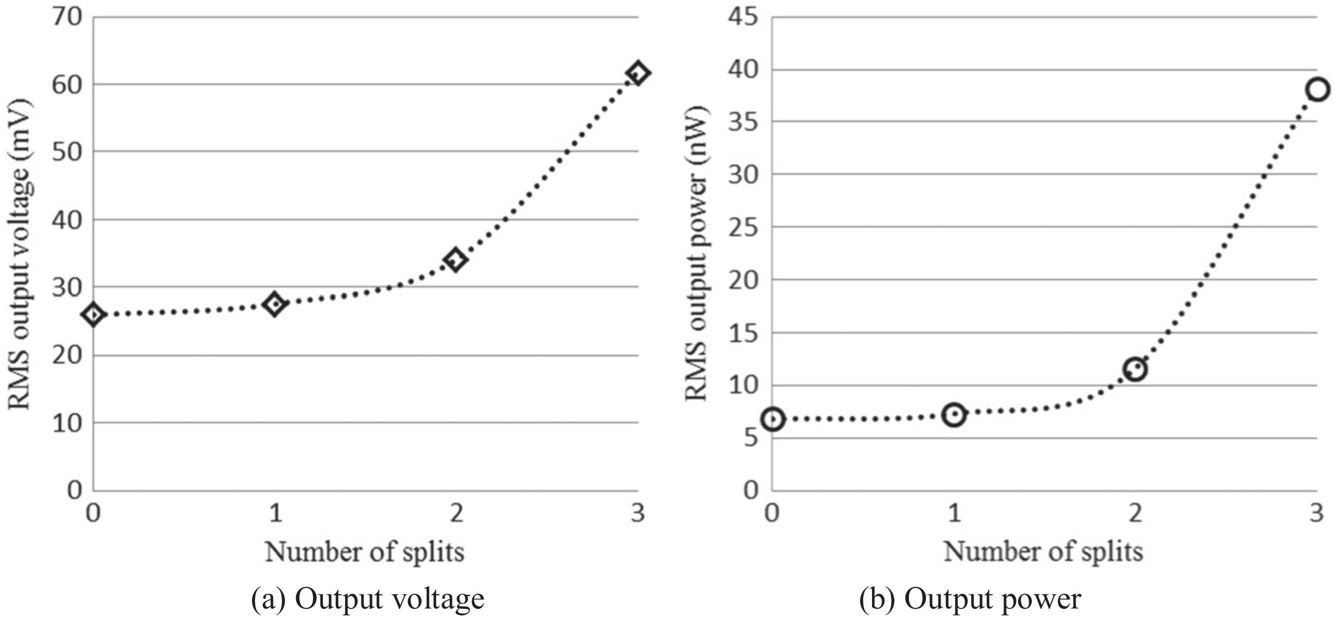



**Figure 6.** Graph showing the RMS voltage output and power output from the piezobeams across a 100 k$\Omega$ resistor for each number of splits. (a) Output voltage (b) Output power.

increased when the piezobeam was split and increased further with higher numbers of splits. At three splits, for example, the harvested voltage was as high as 61.8 mV compared with 26 mV for no split, for an increase of 104.9%. The power output across the 100 k$\Omega$ load increased in a similar manner as the voltage increased. Figure 6 is a graphical visualization of the increase in both the voltage and power outputs, which is shown to occur exponentially. The harvested voltage and power are expected to increase further for higher numbers of splits and commensurate smaller beam widths.

As previously mentioned, both voltage and power outputs increase exponentially with the number of splits, as shown in figure 6. This exponential increment follows the theoretical predictions given in equation (3) for harvested voltage and equation (5) for harvested power across a load. Reduction in the damping of a beam as it splits into smaller elements further ensures an increase in voltage and power in an array of smaller piezobeam elements compared with a single piezobeam of the same total width. The appendix gives further clarification of this assertion. It is expected that there will be further increase in the harvested power output for each split if the load impedance is carefully selected to match the internal impedance of the piezobeam array. However, it is adequate to show the relationship between the number of splits and the power output in this investigation.

## 6. Conclusion

When a beam is made smaller, damping decreases. As the damping decreases, the beam vibrates at higher amplitudes, especially at the resonant frequency. This finding can be used to increase the output from a piezoelectric energy harvester by splitting the piezoelectric cantilever beam into beams of smaller width and then connecting all of them in parallel to form an array of smaller piezoelectric energy harvesters. A substantial increase in harvested power was observed from an array of smaller-width beams compared with a single piezobeam of similar total width.

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**Table 3.** Characteristics of the piezobeams used in the experiment and their corresponding harvested voltage and power across a 100 k$\Omega$ resistor.

| No. of splits | No. of identical piezobeams | Width of identical piezobeams (mm) | RMS voltage output (mV) | Voltage increase (%) | RMS power output (nW) | Power increase (%) |
|---|---|---|---|---|---|---|
| 0 | 1 | 16 | 26.0 | — | 6.8 | — |
| 1 | 2 | 8 | 27.6 | 6.2 | 7.3 | 7.4 |
| 2 | 4 | 4 | 34.1 | 29.3 | 11.6 | 60.0 |
| 3 | 8 | 2 | 61.8 | 104.9 | 38.2 | 229.3 |

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## Appendix

From equation (1), it is clear that A is a constant because it is the thickness ratio between the host structure and the piezo-film. Therefore, in a simpler form, the equation can be rewritten as

$$V = \alpha\gamma\frac{K}{w} \quad \text{(A1)}$$

where $\alpha$ is a constant given by $\frac{-3AB(1-A)g_{31}Ld_o}{ht_b}$ and $\gamma$ is the damping dependency of the beam on its size.

As previously defined, K in equation (A1) is the stiffness of the beam, which can be written in full as

$$K = \frac{8EI}{L^3} \quad \text{(A2)}$$

Clearly, only I, the second moment of inertia, is affected by the changes in the width of the beam, and is given as

$$I = \frac{wt_b{}^3}{12} \quad \text{(A3)}$$

It should be noted that w in equation (A3) is the same as w in equation (A1), which is the width of the beam.

If all these terms are carefully considered, it can be shown that equation (A1) can be further simplified as

$$V = \alpha\beta\gamma \quad \text{(A4)}$$

where

$$\beta = \frac{8E}{12L} \quad \text{(A5)}$$

which is a constant.

It is clear that the voltage output in equation (1) in the paper is a function of only the damping that changes as the width of the beam changes, and not of other parameters.

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