J Nondestruct Eval (2011) 30:91–98
DOI 10.1007/s10921-011-0094-8

Scaling the Operating Deflection Shapes Obtained from Scanning Laser Doppler Vibrometer

Wonki Bae · Yongsoo Kyong · Jedol Dayou · Kyi-hwan Park · Semyung Wang

Published online: 1 March 2011

Abstract Operating Deflection Shapes (ODS) has emerged as one of the powerful techniques in vibration analysis to understand and to evaluate the absolute dynamic behaviour of a machine, component or an entire structure. Traditionally, accelerometers have been used to get the ODS of a structure. However, recent development shows that certain situation may not allow direct contact with the structure under investigation. Therefore, Scanning Laser Doppler Vibrometer (SLDV) has become popular in the investigation. In this paper, a new ODS Frequency Response Function (ODS FRF) for investigations using SLDV is formulated. The ODS FRF is used to construct the ODS of the structure. A new form of scale factor for the ODS FRF is also introduced to normalize the effects from variable excitation force. The importance of this scale factor is demonstrated on a beam and plate under the excitation of varying forces. It is found that the suggested ODS FRF and the scale factor give the desired result in comparison with theory.

Keywords Operating deflection shapes · Scanning laser Doppler vibrometer · Frequency response function · Structural damage detection

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1 Introduction

An operating deflection shapes (ODS) is the deflection shapes of a structure subjected to a single frequency harmonic excitation [1]. When the excitation frequency is close to an isolated natural frequency, the ODS is dominated by the corresponding mode shape, otherwise, the ODS may consist of multiple mode shapes. ODS is therefore shows the deflection of a structure during its operational mode.

ODS analysis has been developed for many applications. These may be for health monitoring of structures, damage detection of bridges, wind turbines and other structures, vibration amplitude assessment and so on [2–4]. This technique has been proven to be useful and currently is still undergoing some developments for more advance applications such as non-destructive analysis or measurements.

Traditionally, accelerometers have been used for ODS analysis. At least two accelerometers are required for this purpose. One of the accelerometers is moved between points of measurements and the other is fixed at a particular point to provide the reference signal [5–8]. This method requires direct contact with the structure under investigation. However, certain condition may not be possible to attach the measuring probes to the structure. Therefore, researchers have turned to other methods of measuring the structural responses to construct the ODS, and one of the intensively investigated means is by using Scanning Laser Doppler Vibrometer (SLDV). SLDV is an attractive tool because it is possible to measure the vibration of a structure remotely without any structural change.

ODS is normally obtained from the ODS frequency response function (FRF). The ODS FRF is given by the multiplication of the auto spectrum of the signal acquired at the measurements point (roving responses) with the cross spectrum between the roving and reference signal [5, 7]. The auto spectrum of the roving response gives the magnitude of the ODS whereas the cross spectrum gives the phase between the measurement and reference points. Therefore, if these magnitudes and phases are combined in a proper way, the ODS of the structure can be constructed.

In the ODS analysis, the type of excitation force must also be considered. In many cases, especially with operating equipment, the measurement signals may be non-stationary (time varying) and the excitation forces cannot be measured. For these cases, different post-processing is required in order to display the ODS from a set of measurements. A means to compensate the effects of the varying forces was introduced and is known as scale factor.

In this paper, a new form of ODS FRF is suggested when SLDV is used in the ODS determination. To account for changes in the excitation level between measurement sets, a new form of scale factor is also introduced. These new form of ODS FRF and scale factor are analyzed theoretically and experimentally.

2 Formulation of the ODS FRF for SLDV

ODS FRF can be measured in several ways as discussed in [9]. Perhaps, the most convenient method is by using single input and single output method. The input provides information regarding the force applied to the structure of interest and the output provides the roving response of points on the structure. However, when the information regarding the excitation force is not possible to measure, for example for a machine under operational conditions, then a reference point can be used as a substitute. From this ODS FRF, ODS of the structure can be constructed.

To construct the ODS, magnitude of the roving points and their phases relative to a reference point are required. Therefore, the output and reference signals acquired during measurements have to be combined in such a way so that information can be extracted from the ODS FRF which is discussed in the following.

ODS FRF can be defined as

$$ODS\,FRF_i = |X_i| e^{j(\phi_i - \phi_{ri})}, \tag{1}$$

where $|X_i|$ is the magnitude of roving signal at the i-th location and $e^{j(\phi_i - \phi_{ri})}$ is the phase between roving signal at the i-th location and reference signal. The magnitude $|X_i|$ corresponding to the response of the i-th point on the structure and can be determined from the auto spectrum of the response signal given by

$$|X_i| = \sqrt{X_i \bullet X_i^*}. \tag{2}$$

On the other hand, the phase between the roving signal and the reference can be determined from the cross spectrum between these signals and is derived as follow. Let X_{ri} be the reference signal for the i-th measurement. If X_i and X_{ri} are defined as, respectively

$$\begin{aligned} X_i &= A_i e^{j\phi_i}, \\ X_{ri} &= B_{ri} e^{j\phi_{ri}}, \end{aligned} \tag{3}$$

where A_i, B_{ri}, ϕ_i and ϕ_{ri} are the amplitude of the point response signal, the amplitude of the reference signal, the instantaneous phase of the point response signal, and the instantaneous phase of the reference signal, respectively, then

$$X_i \bullet X_{ri}^* = A_i B_{ri} e^{j(\phi_i - \phi_{ri})}, \tag{4}$$

and

$$e^{j(\phi_i - \phi_{ri})} = \frac{X_i \bullet X_{ri}^*}{|X_i \bullet X_{ri}^*|} \tag{5}$$

where $|X_i \bullet X_{ri}^*| = A_i B_{ri}$. Therefore, substituting (2) and (5) into (1) gives the ODS FRF as

$$ODS\,FRF_i = \sqrt{X_i \bullet X_i^*} \cdot \frac{X_i \bullet X_{ri}^*}{|X_i \bullet X_{ri}^*|}. \tag{6}$$

For ODS measurements using SLDV, X_i, which is measured by the LDV set, is the frequency spectrum of the velocity of the structure at the i-th point, and it has physical dimension of m/s. On the other hand, if X_{ri} is measured using accelerometer, in normal situation it has dimension of m/s^2. In complete form, the ODS FRF in (6) can be written as

$$ODS\,FRF_i = \sqrt{A_i e^{j\phi_i} \cdot A_i e^{-j\phi_i}} \frac{(A_i e^{j\phi_i} \cdot B_{ri} e^{-j\phi_{ri}})}{|A_i e^{j\phi_i} \cdot B_{ri} e^{-j\phi_{ri}}|}. \tag{7}$$

This equation reduces to

$$ODS\,FRF_i = A_i \frac{A_i B_{ri} e^{j(\phi_i - \phi_{ri})}}{A_i B_{ri}} = A_i e^{j(\phi_i - \phi_{ri})}, \tag{8}$$

where $(\phi_i - \phi_{ri})$ gives the relative phase between the roving point and the reference signals. It can be seen that (8) has physical dimension of [m/s][rad]. However, the ODS FRF derived in (6) has a different form compared to the conventional ODS FRF measured using accelerometers alone [5] which can be written in equivalent form as

$$ODS\,FRF_{ci} = X_i \bullet X_i^* \cdot \frac{X_i \bullet X_{ri}^*}{|X_i \bullet X_{ri}^*|}. \tag{9}$$

The subscript ci denotes the conventional i-th ODS FRF. It should be noted that the magnitude of the ODS FRF given by the first term in (6) has a square root compared to the conventional ODS FRF shown in (9).

To demonstrate the requirement for this new form of ODS FRF, a series of theoretical and actual experiments

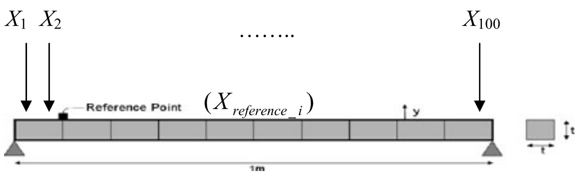


Fig. 1 Theoretical experimental set-up for comparing the ODS derived from the conventional ODS FRF and the new FRF (equation (6))

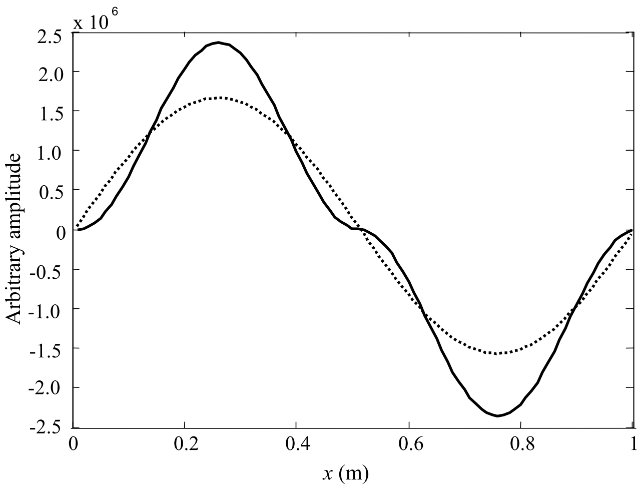


Fig. 2 Theoretical comparison between the ODS of a simply supported beam obtained from the two possible FRFs. *Solid line*—conventional ODS FRF (equation (9)); *dotted line*—equation (6)

were conducted to simulate and to compare the ODS obtained using (6) with the ODS obtained using the conventional equation found in [5]. The theoretical experiments were carried out on a simply supported beam with the following physical characteristics—Young's modulus 2.0×10^{11} N/m^2 and mass density 7850 kg/m^3, with dimensions of 1000 mm $\times$ 5 mm $\times$ 5 mm. With these physical characteristics, the beam has its first three natural frequencies of approximately 12 Hz, 47 Hz and 105 Hz, respectively.

An artificial sinusoidal unit excitation force was applied at $x = 950$ mm, and the frequency of the excitation force was fixed at 47 Hz which is the second natural frequency. Therefore, the ODS has the same shape with the second mode of vibration of the beam. The acceleration at location $x = 100$ mm is used to provide the reference signal (X_{ri}) and the velocity response of the beam is used as an output signal (X_i) with spatial resolution of 10 mm (Fig. 1). Using these information, the ODS obtained using (6) and from the conventional equation were simulated and then compared.

Figure 2 shows the theoretical comparison of the ODS obtained using (6) and using the conventional equation. Obviously, the ODS obtained using the conventional equation does not give the required shape. It shows some distortions especially near the zero-crossing points (nodal point). Similar result was obtained experimentally using a cantilever

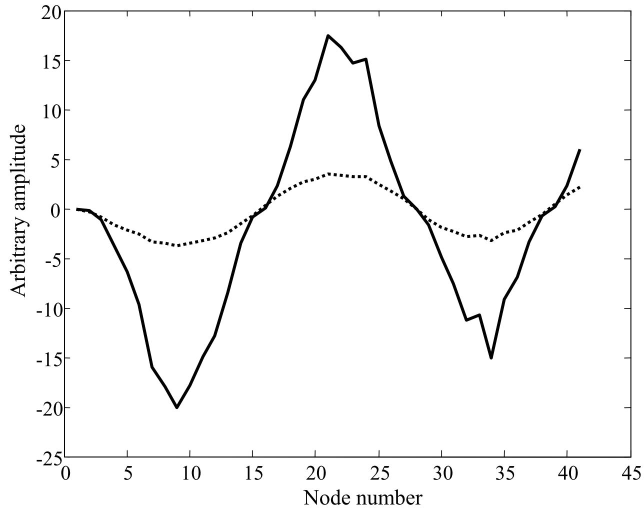


Fig. 3 Experimental comparison between the ODS of a cantilever beam obtained from the two possible FRFs. *Solid line*—conventional ODS FRF (equation (9)); *dotted line*—equation (6)

beam which supports the requirement for the square root in the magnitude of ODS FRF as in (6). This is shown in Fig. 3 and the experimental set-up is similar to Sect. 3.1 for constant force. Therefore, (6) is suggested to be used as the ODS FRF in the future application for SLDV.

3 Formulation of the Scaling Factor of the ODS FRF for SLDV

3.1 The Requirement for ODS FRF Scale Factor

Only under rare condition, the excitation force has constant amplitude that is when the force level is stationary. In this situation, the data acquired from SLDV and accelerometer can be directly substituted into (6) in order to get the ODS FRF and later the ODS. However, during in-operation tests (either during actual operation or during operation-simulation tests), the real loading conditions on the product are present [8] and the load may change from time to time.

The ODS FRF in (6) does not deal with the non-stationary signals; that is proper determination of ODS FRF would not be possible if the excitation force level varies for each measurement. To obtain accurate ODS FRF regardless of excitation force levels, compensation of magnitude in ODS FRF must be considered. To appreciate the requirement for compensation in the ODS FRF, an experiment was carried out on a cantilever beam under two conditions—constant and varying forces. The beam was made of steel with 1 mm thickness, 200 mm length and 25 mm width. A reference accelerometer was attached at $x = 150$ mm, and an exciter type 4809 from Brüel & Kjær which was derived using power amplifier type 2706 from the same company, was used to excite the beam at $x = 195$ mm with constant

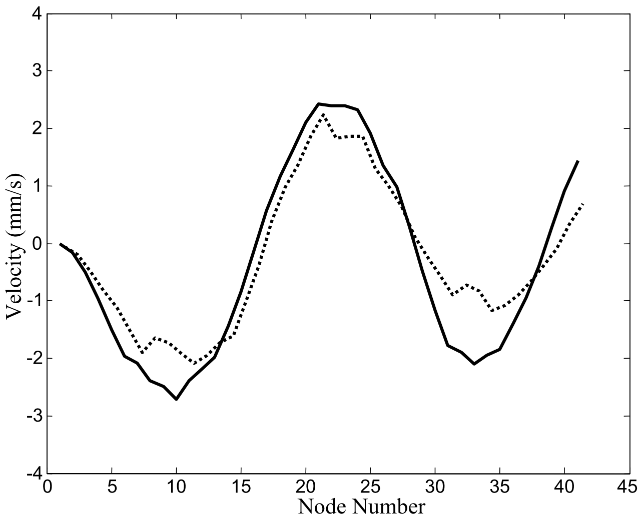


Fig. 4 ODS of the cantilever beam with constant and varying forces amplitude at 614 Hz. *Solid line*—constant force; *dotted line*—varying forces

and varying forces. The accelerometer gives the X_{ri} reference signal for the i-th measurement, and there were 42 number of nodes selected for the SLDV measurements. Each measurement was carried out for 10 seconds and the average value of the measurement in the frequency domain was taken to represent the i-th output signal, X_i. The beam was excited at 614 Hz which is near to its 4th natural frequency. From these data, the ODS obtained using (6) was then simulated for both type of excitations. Prior to this, a modal test was carried out on the beam to determine its natural frequencies and it was found that the first five bending modes has the frequency of 77 Hz, 143 Hz, 327 Hz, 612 Hz and 1066 Hz, respectively. For constant force excitation, the RMS value of the input signal was fixed at 20 mV and for varying excitation forces, the RMS was varied between 10 mV to 30 mV for each point of measurements.

Figure 4 shows the ODS of the cantilever beam obtained from the above experiment. The solid line represents the ODS with constant force and the dotted line represents the ODS with varying forces. It can be seen that the ODS of the beam with varying forces has different shape from that of constant force. For advanced investigation purposes, for example in structural damage detection [3, 10], such a discrepancy might be seen as an existence of damage. Obviously, ODS in (6) has to be corrected to account for changes in the excitation level between measurement sets [7]. This correction is widely known as scale factor.

3.2 Derivation of the ODS FRF Scale Factor

In the previous section, the requirement for scale factor has been discussed. The required scale factor is theoretically derived in this section. For simplicity of discussion, the ODS FRF in (6) is rewritten as

$$ODS\,FRF_i = \sqrt{G_{X_i.X_i}} \cdot \frac{G_{X_i.X_{ri}}}{|G_{X_i.X_{ri}}|}, \tag{10}$$

where $G_{X_i.X_i}$ is the auto spectrum of the roving signal and $G_{X_i.X_{ri}}$ is the cross spectrum between roving and reference signals. Considering the system characteristic of the structure acquired during the measurements, the auto spectrum of the roving signal and the cross spectrum between roving and the reference signals can be expressed as, respectively,

$$G_{X_i.X_i} = \alpha_{xxi}\hat{G}_{X_i.X_i}, \tag{11}$$

$$G_{X_i.X_{ri}} = \alpha_{xri}\hat{G}_{X_i.X_{ri}}. \tag{12}$$

α_{xxi} represents the variable force amplitude in the auto spectrum of the roving signal and α_{xri} represents the variable force amplitude of the cross spectrum signals. Now that the information regarding the excitation force in the signal has been separated, $\hat{G}_{X_i.X_i}$ and $\hat{G}_{X_i.X_{ri}}$ represent only the system characteristics acquired by the SLDV and the accelerometer in the i-th measurement on the structure. Substituting (11) and (12) into (10) gives

$$ODS\,FRF_i = \sqrt{\alpha_{xxi}}\sqrt{\hat{G}_{X_i.X_i}} \cdot \frac{\hat{G}_{X_i.X_{ri}}}{|\hat{G}_{X_i.X_{ri}}|}. \tag{13}$$

Physically, α_{xxi} carries the information of the varying forces experienced by the structure at the roving point for the i-th measurement. At the same time, the reference point also experiencing a varying forces proportional to α_{xxi}. Therefore, α_{xxi} has to be normalized with the varying forces experienced by the structure at the reference point in order to get the correct ODS. If the auto spectrum of the reference signal is written in the similar form as in (11) and (12), then

$$G_{X_{ri}.X_{ri}} = \alpha_{rri}\hat{G}_{X_{ri}.X_{ri}} \tag{14}$$

where α_{rri} represents the variable force amplitude experienced by the structure at the reference point. Dividing (13) with the square root of (14) gives

$$\begin{aligned} ODS\,FRF_i &= \sqrt{\alpha_{xxi}\hat{G}_{X_i.X_i}} \cdot \frac{\hat{G}_{X_i.X_{ri}}}{|\hat{G}_{X_i.X_{ri}}|} \cdot \frac{1}{\sqrt{\alpha_{rri}\hat{G}_{X_{ri}.X_{ri}}}} \\ &= \sqrt{\hat{G}_{X_i.X_i}} \cdot \frac{\hat{G}_{X_i.X_{ri}}}{|\hat{G}_{X_i.X_{ri}}|} \cdot \sqrt{\frac{\alpha_{xxi}}{\alpha_{rri}}} \frac{1}{\sqrt{\hat{G}_{X_{ri}.X_{ri}}}}. \end{aligned} \tag{15}$$

However, in this equation, there is an additional terms in the denominator which is the system characteristic of the structure at the reference point, $\hat{G}_{X_{ri}.X_{ri}}$. From physical dimension point of view, (15) is incorrect. Beside, this term is a redundant information and may affect the result of the ODS.

Therefore, it has to be cancelled out and this can be achieved by multiplying (15) with a global information of the signal from the reference point for all measurements which is the averaged of all signals acquired at the reference point. The global information required for this purpose is given by

$$\sqrt{\frac{\sum_{i=1}^{M}\alpha_{rri}\hat{G}_{X_{ri}.X_{ri}}}{M}} \tag{16}$$

or

$$\sqrt{\bar{\alpha}_{rr}\hat{G}_{X_{ri}.X_{ri}}}, \tag{17}$$

where $\bar{\alpha}_{rr}$ is the average of force amplitude at the reference point for all M measurements. Note that the index i is now removed from the term $\bar{\alpha}_{rr}$ because the averaged force represents the global information of all set of measurements and is no longer an individual characteristic of the i-th measurement. Then, multiplying (15) with (17) leads to

$$ODS\,FRF_i = \sqrt{\hat{G}_{X_i.X_i}} \cdot \frac{\hat{G}_{X_i.X_{ri}}}{|\hat{G}_{X_i.X_{ri}}|} \cdot \sqrt{\bar{\alpha}_{rr}.\frac{\alpha_{xxi}}{\alpha_{rri}}}. \tag{18}$$

It can be seen that the system characteristic $\hat{G}_{X_{ri}.X_{ri}}$ of the structure at the reference point in (15) has been cancelled out.

$\frac{\alpha_{xxi}}{\alpha_{rri}}$ in (18) is the ratio between the force experienced by the roving point and the force experienced by the reference point for the i-th measurement. Based on the physical argument, these forces are proportional to each other. These forces are acquired from the same structure at the same reference of time and therefore a change in any of these terms will result a proportional changes in the other terms. Thus, no matter what the amplitude of the varying forces is, the ratio remains the same. As a result, the effects of the varying forces are normalized.

From the above derivation, the correct scaled ODS FRF of a structure when using SLDV can be written as

$$ODS\,FRF_i = \sqrt{X_i \bullet X_i^*}.\frac{X_i \bullet X_{ri}^*}{|X_i \bullet X_{ri}^*|} \cdot \sqrt{\frac{\sum_{i=1}^{M} X_{ri} \bullet X_{ri}^*}{M \times X_{ri} \bullet X_{ri}^*}}, \tag{19}$$

where

$$\sqrt{\frac{\sum_{i=1}^{M} X_{ri} \bullet X_{ri}^*}{M \times X_{ri} \bullet X_{ri}^*}} \tag{20}$$

is the scale factor of the ODS FRF.

4 Experimental Investigation on the Application of the Scale Factor to ODS FRF

The scale factor to get a correct ODS FRF under the influence of varying forces has been theoretically derived in the previous section. In this section, the application of the newly derived scale factor is investigated experimentally on a beam and also plate. However, detail inspections of (15) shows that the additional terms in the ODS FRF which is

$$\frac{1}{\sqrt{\alpha_{rri}\hat{G}_{X_{ri}.X_{ri}}}} \tag{21}$$

may also be considered as a scale factor. This is because in the equation, the effects from the varying forces are also normalized. Therefore, this equation is included in the investigation.

The structure (beam and plate) were first excited with constant force with RMS value of 20 mV. Later, the RMS of the force amplitude was varied between 10 mV to 30 mV for each point of measurements for the varying excitation forces.

4.1 Experimental Investigation on the ODS FRF Scale Factor on a Beam

In this section, the ODS of a cantilever beam scaled by the newly derived scale factor in (20) and (21) are experimentally investigated. The beam was first excited with varying forces and then its ODS is compared with that of constant force. The same procedure as in the previous section (Sect. 3.1) was followed in order to get the ODS of the beam with constant and varying forces.

Figure 5 shows the scaled ODS of the beam with varying forces in comparison to the ODS of constant force at 614 Hz. It can be seen that the ODS of the beam excited by varying forces with scale factor (20) gives almost identical ODS (shape and amplitudes) to that of constant force. The ODS scaled by (21) gives similar shape with that of constant force but different amplitudes. However, it can be proven that the ODS scaled with (21) is similar to the ODS scaled by (20) if they are normalized to their maximum velocity. Therefore, it can be concluded that both scale factors can be used to compensate the effects of the varying excitation forces and as a result gives the correct deflection shapes of a structure. However, if the deflection amplitude of the structure is one of the main concerns in the investigation then the scale factor in (20) is recommended.

4.2 Experimental Investigation of ODS FRF Scale Factor on a Plate

The investigation on the application of the scale factors is extended on a more complex structure namely a clamped-clamped-free-free steel plate and the results are discussed in this section. A similar procedure as in the previous section was followed and the experimental setup for this purpose is shown in Fig. 6. The plate has a dimensions of $(X, Y, Z) = (100\ \text{mm}, 120\ \text{mm}, 0.6\ \text{mm})$, and the excitation force and reference accelerometer were attached at $(x, y) = (50\ \text{mm}, 20\ \text{mm})$ and $(x, y) = (50\ \text{mm}, 100\ \text{mm})$, respectively. With these conditions, it was found from modal testing that the first four natural frequencies are 149 Hz, 229 Hz, 392 Hz and 490 Hz. A total of 42 velocity measurements (42 number of nodes) of equally spaced point on the plate were acquired using SLDV, and the ODS of the plate with varying forces was simulated with the two scale factors (including the unscaled ODS) and then compared with the ODS of constant force excitation. The frequency of the excitation force was fixed at 231 Hz which is near the second natural frequency of the plate.

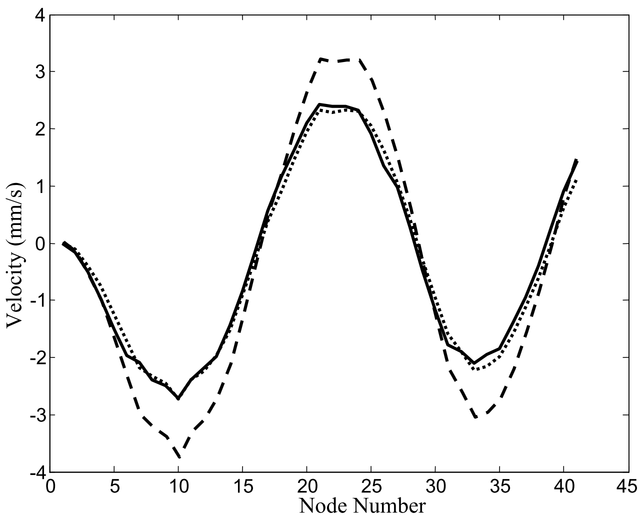


Fig. 5 Comparison of the cantilever beam's ODS with constant force amplitude and scaled ODS with varying forces at 614 Hz. *Solid line*—ODS with constant force; *dotted line* (···)—scaled with (20); *dashed line* (- - -)—scaled with (21)

Figure 7 shows the comparison between the ODS of the plate with constant and varying forces amplitude, scaled and unscaled. It can be seen that the unscaled ODS with varying forces in Fig. 7(b) has different deformation shape compared to the ODS with constant force (Fig. 7(a)). Again, this discrepancy justifies the requirement for a correct scale factor. Detail inspections of the scaled ODS with varying forces in Fig. 7(c) shows that application of the scale factor in (20) gives a similar deformation shape with the ODS of constant force in terms of amplitude and shape. On the other hand, although application of the scale factor in (21) result in similar shape with that of constant force, it has different amplitudes as shown in Fig. 7(d). However, if the ODS from these two different scale factors are normalized to their maximum amplitudes, similar amplitudes and shapes can be obtained, which is also similar to the ODS of constant force. This is clearly shown in Figs. 7(e) and 7(f).

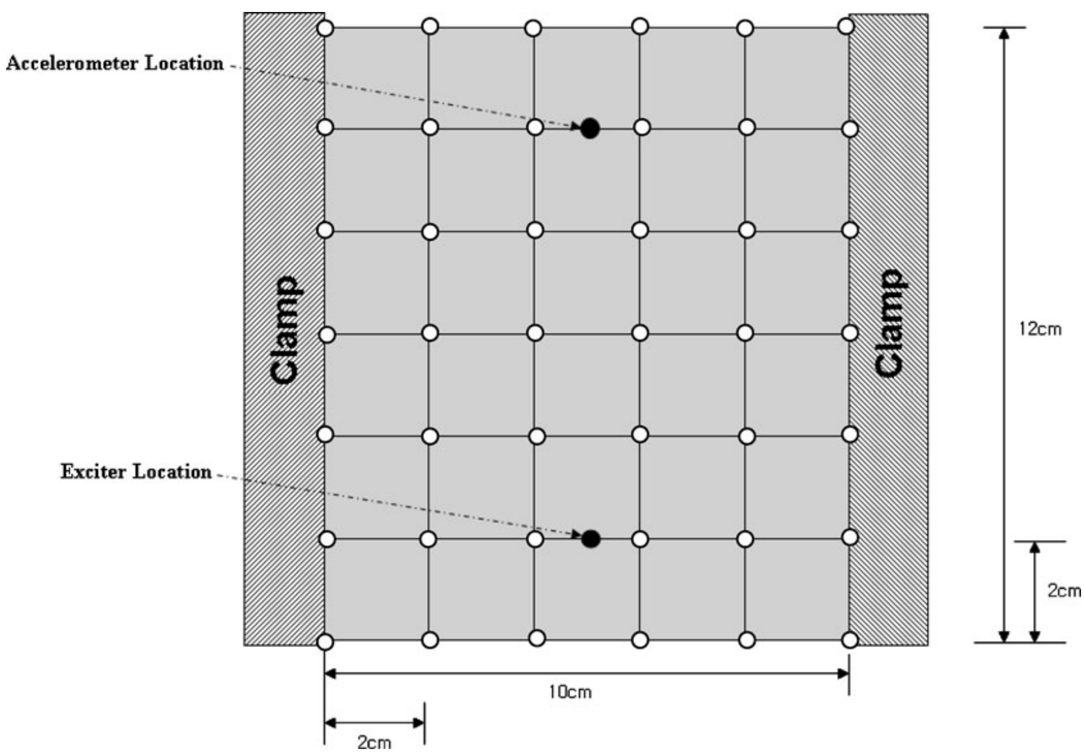
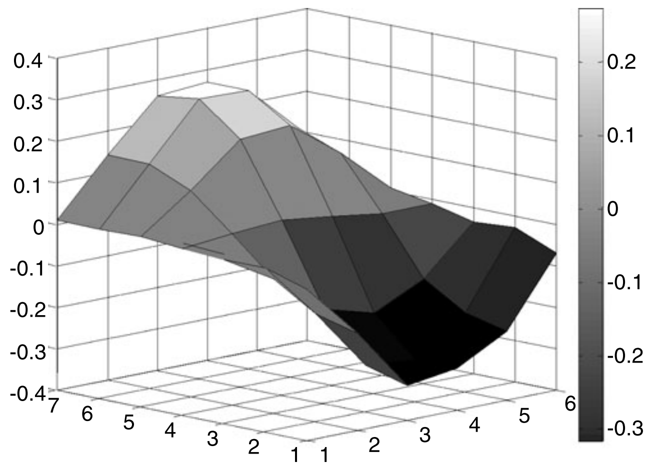
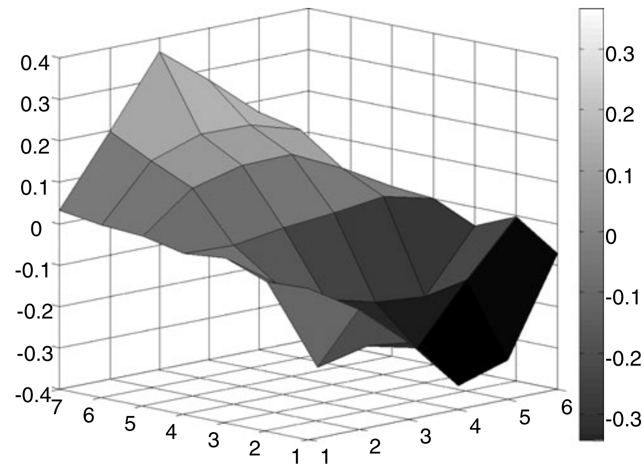


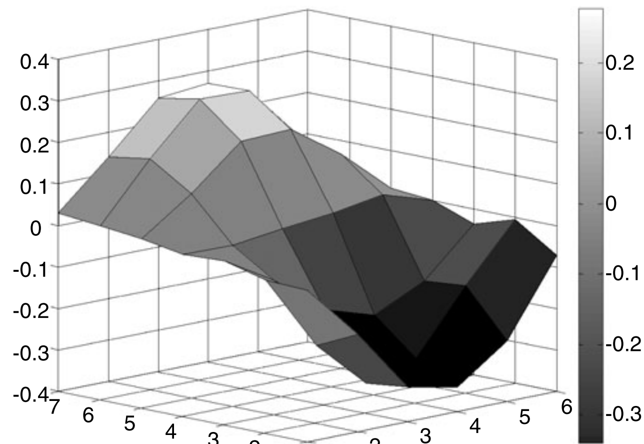
Fig. 6 Experimental set-up for comparing the effects of scaled and unscaled ODS of a plate. ∘ are the measurement points on the plate using SLDV



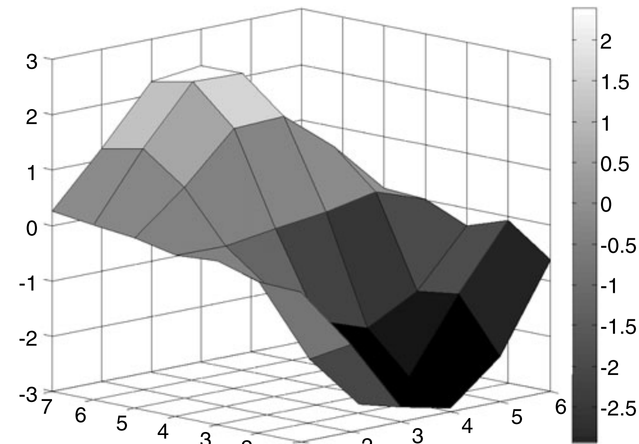
a. ODS with constant force.



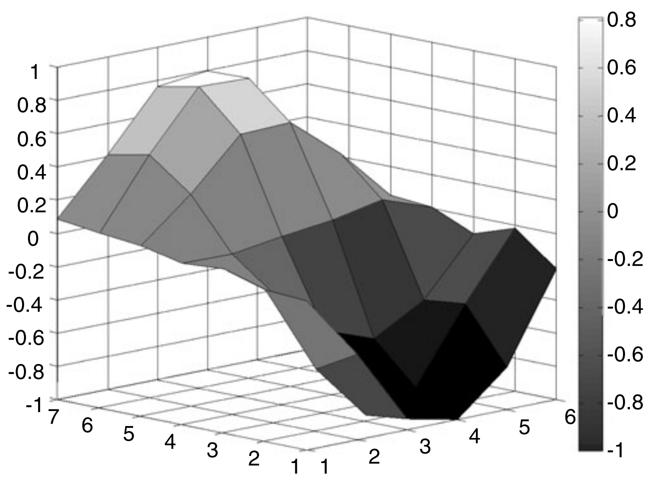
b. Unscaled ODS with varying forces.



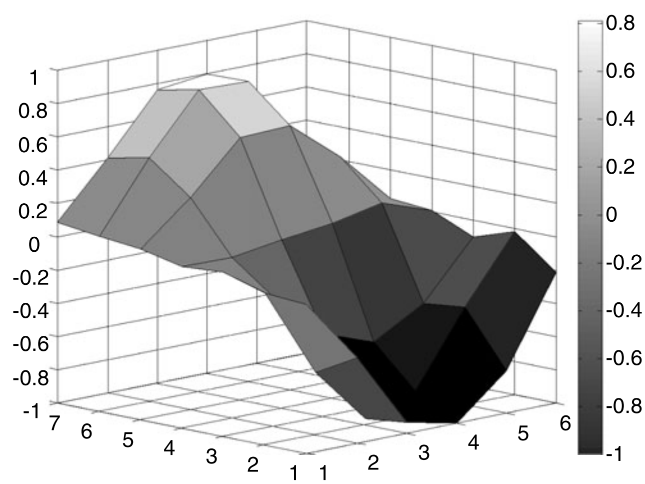
c. Scaled ODS using Equation (20).



d. Scaled ODS using Equation (21).



e. Scaled ODS using Equation (20) - normalized.



f. Scaled ODS using Equation (21) - normalized.

Fig. 7 Comparison between the ODS of the plate with constant and varying forces amplitude, scaled and unscaled

5 Summary and Conclusions

In this paper, a new set of ODS FRF was suggested based on the physical dimension consideration when the ODS of a structure is constructed using SLDV. The new ODS FRF was proven to be correct and gives similar ODS with theoretical prediction. A new scale factor for the ODS FRF to account for changes in the amplitude of the excitation force was also suggested. It was shown experimentally using beam and plate that the new scale factor normalizes the effects of the varying forces. With the new scale factor, similar shape and amplitude of ODS that of constant force can be obtained. The normalization of the effects from the varying forces by the scale factor was explained theoretically.

Acknowledgement This research was supported by Gwangju Institute of Science and Technology (GIST), Republic of Korea.

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